THE LINE OF PROPORTION,

Commonly called

GUNTER'S LINE,

Made Eaſie.

A SECOND PART.

With the addition of other Lines, which
may conveniently be put upon a *Two-foot Rule*,
and their USES Exemplified, In

Arithmetick, *Geometry*, *Military Affairs*, *Trigonometry*,
Aſtronomy, *Dialling*, *Geography*, *Navigation*, &c.

By WIL. LEYBOURN *Philom.*

To which is added a

SUPPLEMENT,

Containing the Deſcription and ſome
Uſes, of a convenient Two-foot
JOYNT-RULE:
Upon which are inſcribed divers Lines and Scales,
ſutable to all ſort of Artificers occaſions.
BY JOHN BROWN.

London, Printed by *W. L.* and *T. J.* for *George*
Sawbridge at the *Bible* on *Ludgate Hill.* 1677.

To the READER.

T*He Good Acceptance which the former Part of this* BOOK *hath received in the World (which was entituled,* The Uſe of the Line of Proportion (*or* Numbers) *commonly called* Gunter's Line made eaſie) *hath animated me to write ſome other* Precepts , *and to adde ſome other* Proportional Lines *of Mr.* Gunter's *firſt contrivance from* *his* Logarithmical Tables *of* Artificial Sines *and* Tangents *upon a* Strait Ruler.

In the Firſt Part *I have principally applied the* Line of Numbers *to ſuch kind of* Menſurations *as are of daily uſe amongſt Workmen, as in the Menſuration of* Board, Glaſs, Timber, Stone, Brick-work, Tiling, Painting, Paving, Plaiſtering, Wainſcotting, &c. *of all which (and ſome other Menſurations) I have given there ſufficient Rules and Examples. Wherefore I ſhall (in this* Second Part) *omit to ſay any thing of ſuch matters or things as I have at large handled therein; although all the*

Work in that Book contained may be performed upon one of the Lines which is upon this Ruler, namely, by the Line of Numbers of two Radiuſſes; *but ſhall principally diſcourſe, or treat, of the Uſes of ſuch other* Proportional Lines *as are inſcribed upon this* Ruler, *as now contrived: And yet I will not forbear to ſhew how to perform many* Problems *in the Former Part by this* Ruler *alſo; but they ſhall only be ſuch, which by the Lines (as they are now diſpoſed) may be wrought with leſs* Trouble, *more* Speed, *and the ſame* Exactneſs; *and many, which there (by the Single Line) required greateſt* trouble *in their* performance,

formance, *may be here done with the greateſt* eaſe; *nay, many (and thoſe the moſt difficult) by inſpection only, not medling with any other; My principal aim in this* Second Part *being to ſhew ſuch other Uſes of the Common (or General) Line* (viz. *the* Line of Numbers) *together with ſuch other Proportional Lines or Scales upon this Ruler inſcribed, in the ſolution of the moſt uſeful and neceſſary Problems in* Arithmetick, Geometry, Aſtronomy, Geography, Navigation, Dialling, Trigonometry, *and ſeveral other of the* Mathematical Sciences, *as ſhall render it a moſt abſolute and neceſſary Concometant, not only for*

for Artificers, *but for all ſorts or degrees of Men, of what quality ſoever, that are any wayes inclinable to, or delighted in* Mathematical Practices.

And in order thereunto I have under apt Heads *and diſtinct Titles (and not miſcellanicuſly) given variety of* Problems *and* Examples *in all the above-mentioned Sciences.*

I ſhall not ſay more to induce you to the peruſal of theſe Tractates, but commend you to the Practice of what is herein contained; and (beſides the delight yon will take therein, the benefit and profit you may re-

receive thereby) will be ſufficient motives to induce you to their peruſal.

And now let me acquaint thee Reader, that unto this Second Part there is added a Supplement, *containing the* Deſcription, *and ſome* Uſes *(and thoſe not a few) of a convenient* Two-foot Joynt-Rule.

Thus have I given you a ſhort Account of what is contained both in this Second Part *and in the* Supplement ; *both which I commend unto thee, wiſhing thee good ſucceſs in thy peruſal and practice of them ; and in a ſhort time thou mayeſt expect ſome other Treatiſes of this kind,* *and of other Parts of the Mathematicks alſo (ſome of them being almoſt ready for the Preſs:) from him who wiſhes thy welfare and the Advancement of Knowledge in the Common-wealth wherein he lives.*

London,
May 21. 1677.

Will. Leybourn.

Advertiſement.

T*Heſe* RULES, *and all other* Mathematical Inſtruments, *either for Sea or Land, are made and ſold by* Walter Hayes *at the Sign of the* Croſs-Daggers *in* Moor-Fields, *near the* Popes-Head Tavern London.

THE LINE OF PROPORTION Made EASIE.

A SECOND PART.

CHAP. I.

The deſcription of the RULER, *and the manner how the ſeveral Lines upon it are to be diſpoſed.*

THe Ruler may be made either of Braſs, Wood or Ivory, and it may be a ſtreight Ruler of two Foot long or more, at pleaſure; or it may be in a ſtreight

B Rule

Rule or Scale but of one foot long, but then ſome of the lines will be very ſhort, and the Diviſion on them too ſmall; Or Thirdly (and beſt of all) upon a Two-foot-Joynt-Rule, which opened will be the ſame as a ſtreight Rule of two-foot long.

The Lines upon the Ruler are in Number Eight, beſides Scales of equal parts, and of Chords, which may be upon the edges of the Ruler. But upon the flat of the Ruler (as I ſaid before) Eight Scales or Lines.

I. The firſt, and uppermoſt is one ſingle line of Numbers, containing the whole length (or very near) of the Rule, divided firſt into ten unequal parts, and thoſe again ſubdivided into ten, ſo often as quantity will permit, according to the uſual manner of dividing of ſuch Lines.

II. Next under this Line (and facing of it,) are three lines of Numbers, all of equal length, and all three of them together, are of equal length to the firſt ſingle line.

III. The third Scale is a Line of Numbers broken, having One in the middle thereof, and broken off at either end of the Rule, at 31 and 62 hundred part,

IV. Underneath this broken Line (and facing of it) is the common line of Numbers of two *Radiuſſes*.

Theſe Four fore-mentioned Lines, ſerve to *Extract* the *Square* and *Cube Roots* by Inſpection, without the uſe of Compaſſes, and for other Uſes alſo, as ſhall hereafter be made manifeſt.

V. The fifth Line is a line of *Artificial Sines*, divided into 90 unequal parts, and ſubdivided. And

VI. Is a line of *Artificial Tangents* numbered unequally to 45 Degrees, and back again towards 90, ſo far as the Ruler will permit.

Theſe two Lines of *Sines* and *Tangents*, are both of them of one Length or *Radius*, and are to be uſed with the fourth line of Numbers of 2 *Radiuſſes*.

VII. The ſeventh Scale is a Line of *Artificial Sines*, having 90 deg. in the middle of the Line, and then the Diviſions are continued up beyond 90 deg. to the end of the Ruler, ending at 84 deg. 10 m. or rather at 174 deg. 10 m.

VIII. The eight Scale is a line of *Artificial Tangents*, which faceth the former line of Sines, having the *Radius* (or 45 deg.) in the middle of the Line, againſt 90 deg. of the *Sines*, and is continued up above 45 deg. to the end of the Ruler;

where

where it terminates at 84 deg. 10 m. as the Sines do, and this avoids backward counting.

Theſe two laſt lines of *Sines* and *Tangents* (being both of the ſame *Radius*) are to be uſed with the Fourth Line of *Numbers* of two *Radiuſſes*, and are of good uſe in the ſolution of *Spherical Triangles*, where Obtuſe Angles are ingredient in the Queſtion: And alſo when the *Tangent* given or required, exceeds 45 degrees.

I ſhall ſay no more concerning the Lines upon the Ruler, for every man being at liberty to inſert ſuch other as his particular occaſion ſhall require, as *Chords*, *Equal parts*, a *Meridian-line*, and ſuch like: In the Figure they are diſpoſed in this Order.

* * *

The USE of the PROPORTIONAL LINES IN *ARITHMETICK.*

CHAP. II.

TO paſs by *Numeration*, *Multiplication*, *Diviſion*, the *Golden Rules* both *Direct* and *Reverſe*, as alſo *Duplicated* and *Triplicated Proportions*, they being ſufficiently treated of in the Seven firſt Chapters of the Firſt part, I ſhall proceed to the work of the Ninth Chapter, which is

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SECTION I.

To Extract the Square Root *by the Lines.*

THe Rule delivered for the Extraction of the Square Root in the Ninth Chapter, is, [*Divide the ſpace between* 1, *and the number whoſe Root is to be Extracted into two eqaal parts, and the middle point ſhall fall upon the Root required.*] So the root of 36 being required, if you divide the ſpace between 1 and 36 into two equal parts, the Compaſs point will reſt upon 6, which is the root of 36.—— Alſo the ſecond *Example* of that Ninth Chapter requires the root of 256, the diſtance between 1 and 256 being divided into two equal parts, the Compaſſes will fall upon 16, the root of 256.

This is the way there preſcribed, and is the only way to perform that Work upon one ſingle Line! But as the lines are now diſpoſed,

If upon the line of Numbers of two *Radiuſes*, you take the diſtance between the middle 1 and 36, the ſame diſtiance ſet upon the 1 at the beginning of the ſame Line ſhall reach to 6 upon the broken Line. Alſo the diſtance between the middle 1 and 256 being applyed to the broken line from 1 in the middle thereof, the point ſhall fall upon 16 in the broken Line, which is the Square Root of 256.

SECTION II.

To Extract the Cube Root *by the Lines.*

THis is the work of the Tenth Chapter, and the Rule there delivered for finding of the Cube Root, is [*Divide the ſpace between*

1 and the Number whoſe Cube Root is required, into 3 equal parts, and the Compaſs point will reſt upon the Root.] So, the Cube Root of 216 being required, the diſtance between 1 and 216 being divided into 3 equal parts, the Compaſs-point will reſt upon 6, which is the Cube Root of 216.—— Alſo another *Example* in that Chapter, is, to find the Cube Root of 1728, which is performed by dividing the ſpace between 1, and 1728 into 3 equal parts, which being done, the Compaſs-point will reſt upon 12, which is the Cube Root of 1728.

This is the way there preſcribed; but by theſe Lines,

If you take the diſtance between the firſt 1 and 216 of the Line of Numbers of 3 *Radiuſſes*, that diſtance will reach from the third 1 of the line of 3 *Radiuſſes* to 6, in the firſt and greateſt line of Numbers.—— Alſo, the diſtance taken upon the Line of Numbers of 3 *Radiuſſes* from the uppermoſt 1 (which repreſents Thouſands) the ſame extent will reach from 1, or 10, at the beginning of the great Line of Numbers, to 12, which is the Cube Root of 1728. —— Likewiſe the diſtance from the firſt 1 in the line of 3 *Radiuſſes* to 729, being taken in the Compaſſes, and applied to the third 1 in the ſame Line, that extent will reach upon the Great Line to 9, which is the Cube Root of 729.

By the diſpoſition of theſe ſeveral Lines of Numbers in this Order, you ſee that both the *Square* and *Cube Roots* may be extracted by once opening of the Compaſſes, but that is not all, for there is yet a greater *Compendium*, for by their being diſpoſed in this Order, both the *Square* and the *Cube* Roots may be *Extracted*, and alſo the Root being given, the Square or the Cube thereof may be found by Inſpection, without the help of the Compaſſes; and how to perform the ſame by theſe Lines, I ſhall now come to ſhew.

SECTION. III.

To extract the Square Root *of any Number by the Lines without Compaſſes.*

THis work is performed upon the Ruler by help of the Line of Numbers of two *Radiuſſes* and the Broken Line. Now when any number is given, and the Square Root thereof is required,

Firſt, you muſt conſider whether the figures in your number given being Integers, be Even or Odd: If your number given conſiſt of even figures (as of 2, 4, 6, or 8) or of odd figures (as 1, 3, 5, 7, or 9)

—— If the number being Integers, conſiſt of even places, then count the number in the firſt *Radius* of the Line of two Radiuſſes; but if it conſiſt of an odd number of Figures, count it in the ſecond Radius of the ſame Line; in either of which caſes (as one of them it will always be) right againſt your number, you ſhall find the *Square Root* thereof; which Root muſt conſiſt of ſo many places as the number admits of *Periods*, or *Points* in the *Arithmetical Extraction.*

Example 1. Let 5276 (a number conſiſting of even places) be given, and let the *Square Root* be required: Count this number in the firſt Radius of the Scale, and right againſt it in the broken Line you ſhall find 72,65 the neareſt *Square Root* thereof.

Example 2. Let 72796, a number of odd places be given, to find the *Square Root* thereof: Count this number in the ſecond Radius of the Line, and right againſt it in the broken Line, you ſhall find 269,8, the *Square Root* thereof; which Root conſiſts of three Figures, becauſe the Sum admits of three Periods or Points; by Extraction by the Pen.

SECTION IV.

The Root being given, to find the Square Number *thereunto belonging.*

THe beſt way to perform this, is by *Multiplication*, and is eaſie enough, but by the *Lines* thus.

1. When the Root given conſiſts of one Figure, as an Integer, ſeek the Integer in the ſingle Line of Numbers once repeated, and right againſt it, on the Line of two Radiuſſes, is the *Square Number*, as it is figured upon the Rule,

2. When the Root conſiſts of two Figures, ſeek it in the ſingle

Line, and right againſt it in the Line of two Radiuſſes, is the *Square Number* belonging to that Root; but two Ciphers or Figures muſt be added to the number found, and that for the reaſon hereafter given.

3. When the Root conſiſts of three Figures, ſeek it (as before) in the ſingle Line, and againſt it, in the Line of two Radiuſſes, you ſhall have the *Square Number* anſwering that Root, by adding four Ciphers or Figures to the number found.

Examples of all theſe Varieties.

1. The ſquare of 2 is 4, the ſquare of 8 is 64, as figured on the Rule.

2. The ſquare of 20 is 400, by adding of two Ciphers; the ſquare of 90 is 1800, by adding two Ciphers to 81, the Line as it is figured.

3. The ſquare of 146, is 21316, by adding of four Figures to 2, the number ſtanding againſt 146:

— And again, The Square of 450 is 202500, by adding of four Figures to 20, the number right againſt it.

Now to more number of Places than theſe, I preſume none will attempt to approach by any Inſtrument how large ſoever; yet this may be a good guide to find the true number of Places: For, Look how many Figures you increaſe the Root more than on the Rule is expreſſed, twice ſo many is to be added to the ſquare: For when 6 on the ſingle Line is called 60, increaſed by one place, then 36 is called 3600, increaſed by two places; and when 7 is called 700, increaſed by two places; then muſt 49 the Root be 490000, increaſed by four places: and thus much ſhall ſerve for the Extraction of the *Square Root*, and finding of a *Square Number* anſwerable to a Root given.

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SECTION. V.

To Extract the Cube Root *of any Number by the Lines, without Compaſſes.*

THis work is performed upon the Ruler by help of the ſingle Line of Numbers and the Line of three Radiuſſes facing the ſame. Now when any number is given, and the *Cube Root* thereof is required, you muſt conſider whether your given number conſiſts of 1,4,7, or 10 Places, or of 2,5, or 8 Places, or of 3, 6 or 9 places: For,

1. If the number do conſiſt of 1,4,7, or 10 places, you muſt count it on the firſt Radius of the Treble Line, and right againſt it on the ſingle Line you have the Root.

2. If the number conſiſt of 2, 5, or 8 places, count it on the ſecond *Radius*: And,

3. If the number conſiſt of 3, 6,

or 9 places, count it in the third Radius, and againſt it on the ſingle Line is the Root required; the Treble Line being figured accordingly from 1 to 1000, of 1, 2 and 3 places.

But the number of Figures in the Root, is according to the number of Points, that the number is capable of in Cubical Extraction by the Pen, the number being pointed at the firſt, fourth, ſeventh and tenth Figures, and ſo forwards, leaving two Figures between Point and Point, in this manner:

$$\underset{.}{5}\ 2\ 7\ \underset{.}{9}\ 1\ 6\ \underset{.}{7}\ 3\ 2\ \underset{.}{5}$$

And ſuch a number as this, will have four Integers in the Root of it.

Examples of all theſe Varieties.

1. The *Cube Root* of 8 is 2, found right over it; 2. The *Cube Root* of 64 is 4, found right over it; 3. The

Cube Root of 729 found in the third Radius, is but 9; becauſe ſuch a number of three places is capable but of one point. Again, 4. The *Cube Root* of 6859 found in the firſt Radius is 19, found right over it, increaſing the Root one Figure, as the *Cube Number*, as to counting on the Line, is increaſed three Figures.

Laſtly, The *Cube Root* of 15625 is 25, of two places increaſed 1, as the *Cube Number* is increaſed 3, as to counting on the Lines.

SECTION VI.

A Cube Root *given, to find the* Cube Number *thereto belonging.*

THis is beſt done by ſquaring the Root given, and multiplying that Product by the Root again, by the Pen, but by the Lines thus as before; the Root being directly found on the ſingle Line, on the treble

Line is the direct anſwer, or *Square Number* required:

But when the Root is increaſed by a Figure, the *Cube Number* muſt be increaſed by three Figures, when the Root is increaſed two Figures, the *Cube Number* muſt be increaſed ſix Figures.

Example, The *Cube* of 2 is 8, the *Cube* of 4 is 64, the *Cube* of 9 is 729 directly, ſo as the Rule gives it.

But the *Cube* of 20, the 2 increaſed by one Figure, is in the treble Line 8000, increaſed three Figures, the *Cube* of 80 is 512000; being three Figures more than 512 the *Cube* of 8.

Again, The *Cube* of 150, being two Figures more than 1 and $\frac{1}{2}$, is increaſed by ſix Figures more than 3 $\frac{375}{1000}$, the Cube of 1 and $\frac{1}{2}$, *viz.* 3375000

The *Cube Root* of a ſmall Number and Fraction is found as any whole Number: *Example*, The *Cube Root* of 38, and $\frac{6}{16}$ is 3 $\frac{28}{100}$ but to pretend to fractions of great *numbers*, by the Line, is to loſe your expectation; but a *Cube Root* may be given to four places, by a good Line in integers, or integers and fractions, and then the *Cube number* will conſiſt of ten places; and ſo much for *Cube Roots* by Inſpection only.

SECTION VII.

Of Reduction *of Vulgar Fractions to Decimal Fractions by the Line.*

The Proportion is,

AS the Denominator of the Vulgar Fraction, is to its Numerator: So is Vnity, or 1, to the Numerator of a Decimal Fraction, equal in Value to the Vulgar Fraction.

Example 1. *What Decimal Fraction is equal to* $\frac{63}{64}$?

Extend the Compaſſes from 84 the Denominor, to 63 the Numerator, the ſame extent will reach from 1 (the ſame way) to 75, or $\frac{75}{100}$. which is a Decimal Fraction equal in Value to $\frac{63}{84}$, 100 being the Denominator.

Example 2. *What Decimal Fraction is equal to* ,$\frac{7}{21}$?

Extend the Compaſſes from 21 to 7, the ſame will reach from 1 to ,33, ſo that 33 hundred parts is equal in Value to ,$\frac{7}{21}$ *fer.*

SECTION VIII.

Of Simple Intereſt *by the Lines.*

Queſtion 1.

IF 100 l. *in* 12 *moneths, do gain* 6 l. *how much ſhall* 326 l. *gain in the ſame time?*

As 100 : is to 6 :: So is 326 : to 19,56.

Etxend

Extend the Compaſſes (on the Line of Numbers of two *Radiuſſes*) from the middle 1, downwards, to 6, the ſame extent will reach from 326, downwards, to 19,56, which is 19 *l.* 11 *s.* 2½ *d.* And ſo much will the Intereſt of 326 *l.* gain in 12 *moneths.* Alſo you may by the ſame work find that 270 *l.* will gain in 12 *moneths*, 16,4 or 16 *l.* 8 *s.*

Queſtion 2.

What is the Intereſt of 175 *l.* 18 *s. and* 11 *d. for a year, at* 6 *l. per Cent. for a year?*

You muſt firſt turn the 175 *l.* 18 *s.* and 11 *d.* into a Decimal Fraction, (either as I have taught in my Arithemetick, or by ſuch a brief Table as this following) and it will be 175,9458 *l.* Which done, extend the Compaſſes from 1, downwards, to 6, the ſame extent will reach, downwards, from 175,9458 to 10,5542; which reduced by the following

Table (or otherwiſe) is 10 *l.* 11 *s.* $1\frac{1}{2}$ *d.* for the Intereſt of 175 *l.* 18 *s.* 11 *d.* for a Year.

A Table to Reduce Engliſh Money into Decimal Parts, and the contrary.

Shill.	*Deci.*	*Shill.*	*Deci.*	*P.*	*Deci.*	*Fart.*	*Deci.*
				11	,0458		
20	100	10	,50	10	,0417	3	,0031
19	,95	9	,45	9	,0375	2	,0021
18	,90	8	,40	8	,0333	1	,0010
17	,85	7	,35	7	,0292		
16	,80	6	,30	6	,0250		
15	,75	5	,25	5	,0208		
14	,70	4	,20	4	,0167		
13	,65	3	,15	3	,0125		
12	,60	2	,10	2	,3083		
11	,55	1	,05	1	,0042		

Queſtion 3.

What will the Intereſt of 784 *l. for* 9 *moneths amount to at* 6 *l. in the* 100 *l. for a Year?*

1. As 100 : is to 6 :: ſo is 784 : to 47,04.

2. As 12 *m.* : is to 47,04 :: ſo is 9 *m.* : to 35,28.

Extend the Compaſſes from 1 to 6, downwards, the ſame extent will reach downwards from 784 to 47,04, which is the Intereſt of 784 *l.* for a year, or 12 moneths.

Again, Extend the Compaſſes from 12 moneths, to 47, 04, the ſame extend will reach, the ſame way, from 9 moneths, to 35,28, which reduced is 35 *l.* 5 *s.* 7 *d.* 1 *q.* almoſt, for the Intereſt of 784 *l.* for 9 moneths.

Queſtion 4.

What will the Intereſt of 328 *l. amount unto in* 20 *dayes, at* 6 *p. Cent for a year?*

1. As 100 : to 6 :: ſo 328 : to 19,68.

2. As 365 : to 19,68 :: So 20 : to 1,075.

Extend the Compaſſes from 100 to 6, the ſame extent will reach, the ſame way from 328 to 19,68, which is the Intereſt of 328 *l.* for a year, or 365 days.

Again, Extend the Compaſſes from 365 days to 19,68, the ſame Extent, the ſame way, will reach from 20 days to 1,075, which is the Intereſt of 328 *l.* for 20 days, and the Fraction reduced is, 1 *l.* 1 *s.* 6 *d.* 3 *q.*

Queſtion 5.

If I receive 60 l. 8 s. and 4 d. for the Intereſt of 755 l. 5 s. for a year, what Intereſt have I allowed in the 100 l. for a year?

Reduce 60 *l.* 8 *s.* 4 *d.* into a Decimal, and it is 60,1616. Alſo reduce 755 *l.* 5 *s.* into a Decimal Fraction, and it is 755,25. Then the Proportion will be,

As 755,25 : to 60,1616 :: So is 100 : to 8.

Extend the Compaſſes from 755, 25, downwards, to 60,16, the ſame extent will reach the ſame way from

100 to 8, which is the Rate of Intereſt that was received.

SECT. IX.

Of Anatociſme *or Compound Intereſt by the Lines.*

Queſtion 1.

To find what the Intereſt of any ſum of Money, with the Principal, will be increaſed unto in any number of years, and at any Rate of Intereſt?

LEt it be required to find unto what ſum of Money 143 *l.* 10 *s.* will be increaſed unto in 10 years, accounting Intereſt upon Intereſt at 6 *l. per. Cent. per Annum* for 100 *l.*

The Proportion is

As 100 *l.*
Is to 106, the increaſe of 100 *l.* in a year,
So is 143 *l.* 10 *s.*

To the increaſe thereof in a year, which being 10 times repeated upon the Line, ſhall at laſt reſt upon the ſum both of Principal and Intereſt in 10 years.

Wherefore,

Extend the Compaſſes from 100, to 106 upon the ſingle Line of Numbers; the ſame extent being ſet to 143 *l.* 10 *s.* (or 143,5) and 10 times repeated upon the Line, the ſame way, the point of the Compaſſes at the tenth remove, ſhall reſt apon 257 *l.* and to ſo much will 143 *l.* 10 *s.* be increaſed to in 10 years.

Queſtion 2.

What is any ſum of Money due any number of years to come, worth in preſent Money; diſcounting intereſt upon intereſt at any rate propoſed?

Let it be required to find what 257 *l.* due at 10 years end, is worth in ready Money, allowing *6 l. per*

Cent. Compound Intereſt. This Problem is but the converſe of the former: For,

The extent of the Compaſſes between 100 and 106, being placed upon 257, and removed backwards ten times; at the tenth remove you ſhall find the Compaſs point to reſt upon 143,5, which is 143 *l.* 10 *s.* and ſo much is the 257 *l.* due at 10 years end, worth in ready Money.

Queſtion 3.

If a Yearly Rent or Anunity, be forborn a certain number of years, what will the arrearages thereof amount unto at the expiration of the time, at any rate of intereſt propoſed?

Suppoſe an Anunity or Rent of 10 *l.* a year be forborn for 4 years, what is the Rent and the arrearages thereof worth at the expiration of the 4 years, allowing 8 *l. per Cent.* profit for the forbearance?

The Proportion is,

(1) As 8 *l.* : is to 100 *l.* :: ſo is 10 *l.* : to 150 *l.*

Extend the Compaſſes from 8 to 100, the ſame extent (the ſame way) will reach from 10, to 125.

Then, by the firſt Queſtion, you may find that 125 *l.* forborn for 4 years at 8 *per Cent.* will be worth 170 *l* from which ſum if you ſubſtract 125 *l.* the remainder will be 45 *l.* and ſo much will the Annuity and the forbearance be worth in ready money, at 8 *per Cent.*

Queſtion 4.

What is a Leaſe or Anunity, to continue a certain number of years, worth in ready money, the Purchaſer propoſing to himſelf a certain Rate of intereſt for the laying out of his money?

Suppoſe a Leaſe to be worth 12 *l.* a year, and of that Leaſe there is

16 years to come: What may be given for this Leaſe, and the buyer have 10 *l. per Cent. per Annum* profit for his Money?

The Proportion is,

As 10 *l.* : is to 100 *l.* Principal : : ſo is 12 *l.* : to 120 *l.* Principal.

Extend the Compaſſes from 10 (upon any of the ſmaller Lines of Numbers) to 100, the ſame extent ſhall reach from 12 to 120; then find by the fore-going Queſtions what 120 *l.* being forborn 16 years, will be worth, which amounts to 551 *l.* Principal and Intereſt; from which ſum take 120 *l.* the Principal, and the remainder will be 431 *l.* the Arrears: Then (by the 2d Queſtion) find what 431 *l.* due after the expiration of 16 years, is worth in preſent Money at 10 *l. per Cent.* and the anſwer will be 93 *l.* 14 *s.* and ſo much is that Leaſe or Annuity for 16 years worth.

Queſtion 5.

What Anunity, to continue any number of years, will a propoſed ſum of money purchaſe, ſo that the Purchaſer may have 10 l. per Cent. *profit for his money laying out?*

Let 500 *l.* be propoſed to be laid out in an Anunity, to continue 16 years, and the Purchaſer will have 10 *l. per Cent.* profit; what will the 500 *l.* purchaſe *per Annum*?

By the laſt Queſtion you found that 93 *l.* 14 *s.* will purchaſe 12 *l.* a year for 16 years, at 10 *per Cent*; this known, the proportion will hold,

As 93,7 : to 12 :: ſo is 500 *l.* : to 64 *l.*

Extend the Compaſſes from 93,7 to 12, the ſame extent will reach the ſame way, from 500 *l.* to 64 *l.* and ſuch an Annunity for 16 years will 500 *l.* purchaſe, and the purchaſer gain 10 *l. per Cent. per Annum* for his Money laying out.

SECT. X.

Of the Rule of Fellowſhip or Company, by the Lines.

Five Men, as A, B, C, D, *and* E, *make a Banck or Stock of Money to trade withal, in all* 300 l. *of which*

		Stock	Gain
A	Put into Stock	84	14
B		72	12
C		48	8
D		54	9
E		42	7
	In all	300	50

By this Stock in a certain time they gain 50 *l.* Now it is required to know how much of this 50 *l.* each perſon muſt have to ballance the Money he put in?

The Proportion is

As 300 *l.* the whole Stock, Is to 50 *l.* the whole gain : So is every Mans ſhare put in, To his particular profit.

Extend the Compaſſes from 300, downwards to 50, the ſame extent will reach from 84 (which was the Stock which *A.* put in) to 14, for the ſhare of *A.* And the ſame extent will reach from 72 the Stock of *B.* to 12 the ſhare of *B*, and ſo the reſt as is above expreſſed.

A Queſtion of Fellowſhip with Time.

Two Merchants accompany, D. *put into Stock* 100 *l. for* 4 *months*, E. *put in* 136 *l. for* 3 *months, and they gained* 50 *l. how much muſt each Merchant have of the profit?*

The Proportion will be

(1) As 1 : is to 4 months :: ſo is 100 *l.* : to 400.

(2) As 1, is to 3 months .: ſo is 136 *l.* : to 408.

Extend the Compaſſes from 1, to 4. the ſame extent will reach from 100, to 400*l*, *A.* his Money and Time multiplied together. Again,

Extend the Compaſſes from 1, to 3, the ſame extent will reach from 136, to 408, *B.* his Time and Money multiplied. The ſum of both is 808.

Then for each Merchants ſhare,

(1) As 808 is to 50 *l.* :: ſo is 400 : to 24,75 for *D.*

(2) As 808 : to 50 *l.* :: ſo is 408 : to 25,25 for *E.*

Extend the Compaſſes from 808 to 50, the ſame extent will reach the ſame way from 400 to 24,75, which is the Gain belonging to *A. D.* and the ſame extent will reach from 408 to 25,25, which is the gains belonging to *E.*

So that {E / D} muſt have {25 05 *s.* / 24 15}

The whole Gain 50 00

The

The USE of the PROPORTIONAL LINES IN GEOMETRY.

CHAP. III.

SECT. I.

Of Superficial and Lineal Meaſures.

I Shall be very brief in theſe kind of Meaſures, as alſo in the ſecond Section, concerning Solid Meaſures, becauſe in the Firſt Part there are plenty of Examples of both kinds; ſo that here I ſhall only give you a Miſcellany of Geometrical Problems, for the farthe illuſtrating the Uſes of the Proportional Lines.

Queſtion 1.

How many foot is contained in a Plank, whoſe breadth is 17 $\frac{1}{2}$ *inches, and* 27 $\frac{1}{4}$ *foot long?*

The Extent from 12 to 17 $\frac{1}{2}$ inches the breadth, will reach from 27 $\frac{1}{4}$ the length in feet, to 39 $\frac{3}{4}$ the Content of the Board in feet.

Or if the breadth and length were given in Foot Meaſure, *then*

The Extent from 1, to 1,46, the breadth, the ſame extent will reach from 27,25 the length, to 39,78, the Content in Feet.

Queſtion 2.

If a Board be 21 $\frac{1}{2}$ *inches broad at one end, and* 16 $\frac{1}{2}$ *at the other end, and* 40 *foot long, how many foot is there contained?*

Adde

Add 21 ½ and 16 ½ together, the ſum is 38; the half whereof is 19, then,

The extent from 12 inches to 19 inches, will reach from 40 the length, to 63 and almoſt a quarter, for the Content.

What is here ſaid of *Board*, is to be underſtood of *Paving*, *Plaiſtering*, *Flooring*, *Wainſcotting*, or any other *Superficial Meaſure*; as I have elſewhere exemplified.

Queſtion 3.

If a Brick Wall be 37 *foot long*, 26 ½ *foot high*, *and* 3 ½ *Bricks thick*, *how many Rod doth this Wall contain*, *it being reduced to one Brick and half thick*, *which is the Standard thickneſs for Brick-work?*

(1) As 1 is to 29 ½ the height, ſo is 37 the length, to 980 ½ the Superficial Content.

(2) As 272 ¼ (the number of feet in one Rod , is to 1 ; ſo is 980 ½, to 3 Rod ½ , and 14 parts over.

Extend the Compaſſes from 1 to 26 ½ the height , the ſame extent will reach from 37 the length to 980,5. Again,

Extend the Compaſſes from 272 ¼ the Feet in one Rod, downwards to 1 ; the ſame extent will reach from 980,5 downwards to 3,5 Rod , and 14 Decimal parts over : Wherefore ſet one Foot of the ſame extent of the Compaſſes in 14, and the other ſhall reach upwards to 38 foot , ſo that the Wall contains 3 Rod ½ and 38 Foot.

But being the Wall is 3 ½ Bricks thick , and it muſt be reduced to one Brick and half thick , Take this general proportion for the reducing Brick-work from any thickneſs to Brick and half.

As 3
Is to the number of half Bricks that any Wall is in thickneſs (as here 7.)
So is the number of Feet contained in the Superficies of tthe Wall (as here $980\frac{1}{2}$.)
To the number of Feet that the Wall contains, being reduced to one Brick and half (as here 2286.)

Again for the Rods,

As $272\frac{1}{4}$ is to 1: So is 2286, to $8\frac{1}{4}$ Rod and 42 Foot.

(1) Extend the Compaſſes from 3 to 37, the ſame extent will reach from 980,5 to 2286.

(2) Extend the Compaſſes from $272\frac{1}{4}$ downwards to 1, the ſame extent will reach the ſame way from 2286 to $8\frac{1}{4}$ Rod and 15 Decimal parts over; wherefore ſet one foot of the ſame extent in 15, and the

other ſhall reach upwards to 42 foot; ſo the whole Wall reduced is 8 $\frac{1}{4}$ Rod 42 Foot.

Queſtion 4.

How to find the Aera of a Regular Poligon of 5,6,7,8, or any other number of Sides.

Meaſure the diſtance from the Center to the middle of one of the Sides, which ſuppoſe 32 inches, and let the ſide of the Poligon be 24 inches, and let there be 8 ſides in the Poligon: that is 192 inches in all the ſides, the half is 96, then ſay:

As 1 is to 32 the Perpendicular, ſo is 96 the half Perimiter of the Poligon, to 3072 the Area of the whole Poligon.

Extend the Compaſſes from 1 to 32, the ſame will reach from 96 to 3072.

Queſtion

Queſtion. 5.

If the Diameter of a Circle be 14 *inches, and the Area thereof* 154 *inches, what ſhall be the Area of another Circle, whoſe Diameter is* 28 *inches?*

Out of the broken Line, take the diſtance between 14 and 28 (the 2 Diameters) the ſame extent ſhall reach upon the ſame Line of 2 Radiuſſes, from 154 the leſſer Area, to 616 the Area of the Circle, whoſe Diameter is 28.

Queſtion 6.

If a Piece of Land that is 20 *Pole ſquare, be worth* 30 l. *what ſhall another piece of Land of the ſame goodneſs be worth, that is* 35 *Pole ſquare?*

Take the diſtance between 20 and 35, out of the broken Line (but becauſe you cannot conveniently take it from that, take it out of the ſingle Line) that diſtance applied to the

Line of 2 Radiuſſes, will reach from 30, to 91,8, that is 91 *l.* 16 *s.* and ſo much will the piece of Land be worth, whoſe Side is 35 Pole.

Queſtion 7.

If the Area or Content of a Circle be 154, *whoſe Diameter is* 14, *what ſhall the length of the Diameter of that Circle be, whoſe Area is* 616*?*

Out of the Line of two Radiuſſes, take the diſtance between 154 and 616, the ſame extent applied to the ſingle Line, ſhall reach from 14 to 28, the Diameter of the greater, Circle.

Queſtion 8.

There is a piece of Land 20 *Pole ſquare, and is worth* 30 l. *and there is another piece of the ſame Land worth* 91 l. 16 s. *how many Pole ſquare ought that piece of Land to be?*

Out of the Line of 2 Radiuſſes

take the diſtance between 30, and 91,8, the ſame diſtance applied to the broken Line (or to the ſingle Line) will reach from 20 to 35; and ſo many Pole ſquare ought the other piece of Land to be.

Queſtion 9.

How many Acres of Land of Engliſh meaſure (16,5 foot going to the Pole) are contained in 30 Acres of Iriſh meaſure, where 21 foot goes to the Pole?

Take out of the broken Line the diſtance between 16,5 and 21, the ſame extent will reach from 30 to 48,6 upon the Line of 2 Radiuſſes, and ſo many Engliſh Acres are contained in 30 Iriſh Acres.

Queſtion 10.

The Baſe 65, and Perpendicular 76, of a Right-an led Triangle, being given, to find the Hypothenuſe?

Seek 65 and 76 in the broken Line, and right againſt 65 you ſhall find 4225, and againſt 76 you find 5776 in the Line of 2 Radiuſſes; theſe added together make 10001 which found in the Line of 2 Radiuſſes, againſt it in the broken Line, you ſhall find 100 and ſomewhat more for the length of the Perpendicular.

Queſtion 11.

The Hypothenuſe 100, *and one Side* 65, *of a Right-angled Triangle, given, to find the other Side?*

Seek 100, and 65 in the broken Line, and in the Line of 2 Radiuſſes you ſhall find 10000 againſt 100, and 4225 againſt 65: Subſtract 4225 from 10000, and the remainder will be 5775, which found in the Line of 2 Radiuſſes, againſt it in the broken Line, you ſhall find 76 for the length of the other ſide.

Queſtion 12.

The three Sides of an Obtuſe-angled Triangle are 80, 40, *and the longeſt ſide ſubtending the Obtuſe Angle is* 100, *how far without the Obtuſe ſhall the Perpendicular fall?*

Find 80 and 40 in the broken Line, againſt which in the Line of 2 Radiuſſes, you ſhall find 6400 and 1600; theſe two added make 8000, which ſubſtracted from 10000 the Square of the third ſide 100, there remains 2009, the half whereof is 1000, and that divided by 40, gives 25 for the diſtance of the Perpendicular from the Obtuſe Angle.

Queſtion 13.

The ſame three Sides given, as before, 80, 40, *and* 100, *to find where the Perpendicular ſhall fall from the Obtuſe Angle?*

Find 80 and 100 in the broken Line, againſt which in the Line of 2 Radiuſſes you ſhall find 6400, and 10000 which make 16400, from which ſubſtract 1600, the ſquare of 40, the third Side, and the remainder will be 14800, the half whereof is 7400, which divided by 100, the Quotient will be 74, the length upon the Baſe where the Perpendicular will fall.

Queſtion 14.

The three ſides of any Triangle 100, 80, *and* 40 *being known, to find the Area, without knowing of the Perpendicular?*

Firſt add the three ſides together, their ſum is 220, the half ſum 110, from this half ſum ſubſtract each ſide ſeverally, and the ſeveral diſtances will be 10, 30, and 70. Multiply any 2 of theſe differences together, and the product of them by the third difference,

ference, ſo ſhall the laſt Product be 21000, which multiply by the half ſum 110, and that Product will be 2310000, whoſe Square Root is the Area of the Triangle. Wherefore, take (or count) 2310000 out of the Line of 2 Radiuſſes, and that extent will reach upon the ſingle Line, from the beginning to 1520, the Root of 2310000, equal to the Area of the Triangle.

Queſtion 15.

The two Diameters of an Ellipſis 32 *and* 24, *being given, to find the Area of the Ellipſis?*

Extend the Compaſſes from 1 to either of the Diameters (as 32) the ſame extent ſhall reach from 24 (the other Diameter) to 768 the Area of a Rectangle Figure, made of the two Diameters: Then,

As 100 : to 78,54 ::
So is 768, to 603,19

Extend the Compaſſes from 100, to 78,54 (a fixed Area) the ſame will reach from 768 (the Rectangled Figure made of the two Diameters) to 603,19, the Area of the Ellipſis.

Queſtion 16.

To find the Diameter of a Circle whoſe Area ſhall be equal to the Area of the former Ellipſis?

Upon the Line of Numbers of two Radiuſſes, open the Compaſſes from 24 to 32 the two Diameters of the Ellipſis, that diſtance applied to the ſingle Line, will reach from 24 the leſſer Diameter, to 27,71 the Diameter of a Circle, whoſe Area ſhall be equal to the Area of the Ellipſis.

Queſtion

Queſt. 17.

The Chord Line 60,8, and Altitude 14, of the Segment of any Circle, being known, to find out the other parts of the Circle and the Area of the Circle?

1. Extend the Compaſſes from 1 to 30,4, half the Chord of the Arch, and that diſtance again repeated from 30,4, will reach to 924,16, the ſquare of half the Arch Line.

2. Extend the Compaſſes from 14 (the Altitude of the Arch) to 1, the ſame will reach from 924,16, to 66, to which if you adde 14 the Altitude of the Arch, the ſum will be 80, for the Diameter of the Circle, the half whereof 40, is the Radius of the Circle.

3. Adde half the Segments Chord 30,4, and the Segments Altitude 14, together, they make 44,4, whoſe

Square Root is 6,67 *fere*, and is the length of the Chord of half the Segments Arch.

SECT. II.

Of Solid Meaſures.

Queſt. 1.

If a piece of Square Timber be 15 inches broad, 22 inches deep, and 20 foot long: how many ſol d foot are contained therein?

Extend the Compaſſes upon the Line of two Radiuſſes, from 15 inches the breadth, to 22 in the depth, that extent ſhall reach from 15 upon the ſingle Line, to 18¼ inches, for the true ſquare at the end, then your proportion will be

As 12 inches,
 To the inches ſquare 18¼,

So is the length in feet 20,

To a fourth ; and that fourth to 46 foot.

Extend the Compaſſes from 12 to $18\frac{1}{4}$, the ſame will reach from 20 the length , at twice turning the Compaſſes, to 46 , the quantity of feet contained in the whole piece.

Or in Foot Meaſurs.

Extend the Compaſſes from 1,25, the breadth, to 1,84 the depth, upon the Line of two Radiuſſes, that diſtance applied to the ſingle line, ſhall reach from 1, 25 to 1,52.

Again , Extend the Compaſſes from 1 to 1,52 , the ſame extent ſhall reach from 20 , at twice turning of the Compaſſes , to 46 the content of the Piece in Feet.

Queſtion 2.

If a Piece of Tapering Timber be 2,2 *foot, and* 0,41 *foot at one end, and* 1,32 *foot, and* 1,75 *foot at the other end, and* 12 *foot long; how many ſolid foot is contained in this Piece of Timber?*

1. Upon the Line of two Radiuſſes, take the diſtance between 1, and 0,41, the ſame extent will reach downwards from 2,2, to 0,90, for the content of the Baſe at the little end.

2. Upon the ſame Line take the diſtance between 1 and 1,32, the ſame extent will reach from 1,75, to 2,31, the content of the greater end.

3. Extend the Compaſſes from 1, downwards to 90, the Area of the leſſer Baſe, the ſame extent will reach from 2,31, the Area of the greater Baſe, to 2,08, the product

of the two ends multiplied together; the Square Root whereof is 1,44 : Add this Root and the two Baſes together, their ſum is 4,65. Then again,

$$\begin{array}{r} 0{,}90 \\ 2{,}31 \\ \underline{1{,}44} \\ 4{,}65 \end{array}$$

Extend the Compaſſes from 1 to 4 (which is one third of the length of the piece) the ſame extent ſhall reach from 4,65, to 18,60 the true content of the whole piece.

Queſtion 3.

If a Cube, whoſe ſide is 12 *inches, doth contain* 1728 *Cubical inches, how many Cubical inches ſhall a Cube contain, whoſe ſide is* 8 *inches?*

Out of one of the Lines of 3 Radiuſſes take the diſtance from 12 to 8, the difference of ſides, that ſame diſtance applied to the ſingle Line, ſhall reach from 1728 downwards to 512, the ſolid inches in a Cube, whoſe ſide is 8 inches.

Queſtion 4.

If a Bullet, or Sphear, being 6 inches Diameter, do weigh 30 l. *what ſhall a Sphear of the ſame metal weigh, whoſe Diameter is 7 inches?*

Take the diſtance between 6 and 7 out of the Line of 3 Radiuſſes, the ſame extent applied to the ſingle line, will reach from 30, to 47,7, and ſo much will a Bullet of the ſame metal weigh, whoſe diameter is 7 inches.

Queſtion 5.

If a Ship of 300 *Tun burthen, be* 75 *foot by the Keel, what burthen ſhall that Ship be, whoſe Keel is* 100 *foot?*

The diſtance between 75 and 100, being taken out of the Line of three Radiuſſes, applied to the ſingle Line, will reach from 300 Tun, to 713 the burthen of that Ship, whoſe Keel is 100 foot.

Queſt.

Queſtion 6.

If a Ship of 300 Tun, be 29,5 foot at the Beam; what ſhall the length of the Beam of that Ship be, whoſe burthen is 713 Tun?

Out of the ſingle Line, take the diſtance between 300 and 713, that ſame extent applied to the Line of three Radiuſſes, ſhall reach from 29,5, to 39,35, for the length of the Beam of a Ship, whoſe burthen ſhall be 713 Tun.

Queſtion 7.

If a Ship of 300 Tun be 13 foot in Hold, what ſhall that Ship be in Hold, whoſe burthen is 713 Tun?

Out of the ſingle Line, take the diſtance between 300 and 713, that diſtance applied to the Line of three Radiuſſes, ſhall reach from 13, to 17,35, and ſo much ſhall that Ship be in Hold, whoſe burthen is 713 Tun.

Queſtion 8.

If a Braſs Piece of Ordnance, whoſe Diameter is 11,5 *inches, do weigh* 1900 *pounds, what ſhall another Piece weigh, (of the ſame ſhape) whoſe Diameter is* 8,75 *inches?*

The Extent between 11,5 and 87,5 taken upon the broken Line of three Radiuſſes, will reach upon the ſingle Line, from 1900 to 837; and ſo much ſhall that Piece weigh, whoſe Diameter at Bore is 8,75.

The USE of the PROPORTIONAL LINES IN *Military Affairs*.

CHAP. IV.

SECT. I.

Queſt. I.

How to order any number of Soldiers into a Square Battail; ſo that there ſhall be as many in Rank as in File?

LET it be required to make a Square Battail of 2704 men, ſo that there be as many in Rank as in File.

Foraſmuch as the number of Souldiers do conſiſt of an even number of Figures, ſeek that number 2704, in the firſt Radius of the Double Line of Numbers, and right againſt it in the Broken Line, you ſhall find 52, and ſo many muſt be in Rank, and as many in File: And theſe Souldiers, if they be imbattelled at Order (which is 3 Foot in Rank and as much in File) then will they occupy 24336 ſquare foot of Ground; which by the Lines you may thus find.

Extend the Compaſſes from 1 to 3 (the diſtance in Rank and File) the ſame extent will reach from 52 to 156; find 156 upon the Broken Line, and againſt it in the Double Line you ſhall find 24336, the Ground that theſe Souldiers will occupy, being at their Order of 3 foot.

Queſt.

Queſt. 2.

Any number of men being propoſed, to place them in Battalia, in ſuch order that there may be as many more in Rank as in File, and that they may ſtand at Cloſe Order, which is $1\frac{1}{2}$ foot?

LEt the number given be 2602, count the half thereof 1301, upon the Double Line, and againſt it you ſhall find in the Broken Line 36, which is the depth in File, and then there muſt be 72 in Rank, which is twice 36.

Now for the Ground that theſe will occupy, being at Cloſe Order,

As 1 : is to 1,5 : : ſo is 36 : to 54: and ſo is 72 to 1080.

Extend the Compaſſes from 1 to 1,5, the ſame extent will reach from 36 (the depth of men in File) to 54 the ſide of the Ground. ---- Again, the ſame extent will reach from 72 (the Front of the men in Rank, to 1080 the length of the Ground. — Then for the Area,

Extend the Compaſſes from 1 to 1080, the ſame extent will reach from 54 to 58320, and ſo many ſquare foot of Ground will theſe 2602 men occupy at Cloſe Order.

Queſt. 3.

Any number of men being propoſed to be put in Battalia, and a certain number named to be either in Rank or File, to find the other number?

LEt it be required to place 872 men in Battalia, ſo that there ſhall be 8 in File, how many muſt there be in Rank; or how many Files muſt there be?

The Proportion to work this is,

As 8, the depth in File,
Is to 872, the number of Souldiers,

So is 1, to 109 the number of men in Rank.

Extend the Compaſſes from 3, to 87, the ſame extent (the ſame way) will reach from 1, to 109.

Queſt. 4.

Any number of Souldiers being given, together with their diſtance in Rank and File, to order them into a Square Battail of Ground?

LEt the number of Souldiers given be 3000, their diſtance in File 7 foot, and in Rank 3 foot;

The Proportion holds,

As 7: to 3 : : ſo is 3000: to 1286.

Extend the Compaſſes from 7, downwards to 3, the ſame extent will reach from 3000, downwards to 1286.

Seek 1286 in the firſt Radius of the Broken Line, and juſt againſt it you ſhall find 35,7, the number of men to be placed in File — 35 men is too little and 36 men will be too much; but men are not to be divided in parts.

Queſt. 5.

How to order any number of ſouldiers into Rank and File, ſo, that their diſtance in Rank ſhall be to the diſtance in File, in ſuch proportion as any two numbers given are?

IF 3000 ſouldiers were to be ordered in Rank and File, ſo that the diſtance between Rank and Rank ſhall be in proportion to the diſtance between File and File, as 5 is to 9 (that is) if the men in File ſtand 9 foot aſunder, the men in Rank ſhall ſtand 5 foot aſunder.

The Proportion is,

As 5 : to 9 :: ſo is 3000 : to 5400.

Extend the Compaſſes from 5 to

9, the ſame extent will reach from 3000 to 5400 — Seek 5400 in the Double Line of Numbers, and againſt it in the Broken Line, you ſhall find 73,5, for the number of men in Rank. — Then for the number of men in File,

As 73,5: is to 1 :: ſo is 3000 to 41 *fere.*

Extend the Compaſſes from 73,5 to 1, the ſame extent will reach from 3000 to 41, and ſo many men muſt be in File — But here the number of men are 3013, which 13 over muſt be ſupplied, or elſe 28 men muſt be taken off and diſpoſed of as Scouts, Centinels, or the like; otherwiſe there muſt be one File leſs.

Queſt. 6.

There are 8100 *in a ſquare Battail drawn up, and it is required to have 6 Ranks of Pikes to arme the ſame ſquare Body round about; how many Ranks muſt there be in the whole ſquare Battail, and what number of Pikes and what of Musketeers?*

THe Square Root of 8100 is 90, the number of men in Rank and File; now for that there muſt be 6 Ranks of Pikes about the Muſketeers, there will be 12 Ranks leſs of them both in Front and Flank, than in the whole Body: wherefore ſubtract 12 from 90, there will remain 78, which number find in the Broken Line of Numbers, and right againſt it you ſhall find 6084, the number of Musketeers, and that taken from 8100, there remains 2061, for the number of Pikes.

SECT. II.

Concerning the Quartering of Souldiers by the Lines.

Queſt. 1.

If 1000 Souldiers may be lodged or quartered in a ſquare of 300 foot of Ground, how many foot long muſt the ſide of a ſquare be, that the Ground included may lodge 5000?

EXtend the Compaſſes from 1 to 300 (the ſide of the Square which will lodge 1000 Souldiers) the ſame extent will reach forward from 300 to 90000, then

The Proportion will be

As 1000 : is to 5000 :: ſo is 90000: to 450000.

Seek this number in the Double Line of Numbers, and againſt it in the Broken Line you ſhall find 671, and ſo much muſt the ſide of a Square be, that muſt lodge 5000 Souldiers with the ſame convenience that 1000 Souldiers were lodged in a Square whoſe ſide was 300 foot.

According to this Method may all Queſtions of this kind be reſolved.

The

The USE of the
PROPORTIONAL
LINES
IN
TRIGONOMETRY:
OR,
The Menſuration of Triangles
BOTH
Plain and *Spherical.*

CHAP. V.

Definitions and Theorems Trigonometrical.

1. A Triangle is a Figure conſiſting of three Sides and as many Angles; as is the Triangle CAB, in *Fig.* I.

2. Any two Sides of a Triangle are called the Sides of the Angle contained by them; as the Sides CB and AB, are the Sides containing the Angle CBA.

3. The meaſure of an Angle is the quantity of the Arch of a Circle, deſcribed upon the angular Point, and cutting both the Sides containing the Angle.

4. A Degree is the 360 part of any Circle. Therefore,

5. A Semicircle contains 180 degrees. And

6. A Quadrant (or right Angle) contains 90 degrees.

7. The Complement of an Angle leſs than 90 degrees, is ſo much as that Angle wanteth of 90 degrees.

8. The Complement of an Angle to a Semicircle, is ſo much as that Angle wanteth of 180 degrees.

9. An Angle is either Right, Acute, or Obtuſe.

10. A Right Angle is that whoſe

meaſure is 90 degrees, or a Quadrant.

11. An Acute Angle is leſs than a right Angle, and alwayes contains leſs than 90 degrees.

12. An Obtuſe Angle is greater than a right Angle, and alwayes contains more than 90 degrees.

13. A Triangle is either right-angled or oblique-angled.

14. A right-angled Triangle is ſuch a Triangle as hath one right Angle. As the Triangle C A B, (*Fig. I.*) hath one right Angle, namely, that at A, which containeth juſt 90 degrees.

15. In every right-angled Triangle. that Side which ſubtendeth (or lieth oppoſite to) the right Angle is called the ***Hypotenuſe***; and of the other two Sides, the one is called the ***Perpendicular***, and the other the ***Baſe***, at pleaſure: But moſt commonly the ſhorter ſide is called the Perpendicular, and the longer the

Baſe. Thus in the Triangle C B A, B C is the Hypotenuſe, C A the Perpendicular, and A B the Baſe.

16. In every right-angled Triangle, if you have one of the acute Angles given, the other is alſo given, it being the Complement thereof to 90 degr. As in the Triangle C A B, if you have the Angle at C 53 degr. 7 min. given, you have alſo the Angle at B given, it being the Complement of that at C to 90 degr. wherefore take 53 degr. 7 min. from 90 degrees, and there will remain 36 degr. 53 min, which is the quantity of the Angle at B.

17. In all right-lined Triangles whatſoever (either right-angled or oblique-angled) the three Angles together are equal to two right Angles, or contain 180 degrees: Therefore, if you have any two Angles of a Triangle given, you have alſo the third given, it being the Complement of the other two to 180 degrees:

grees: Thus, in the Triangle CDB, *Fig.II.* if there were given the Angle CDB, 43 deg. 20 min. and the Angle CBD 14 degrees 40 min. I ſay, by conſequence you have the third Angle DCB alſo given, it being the Complement of the other two to 180 degr. For the two given Angles BDC 43 degr. 20 min. and CBD 14 degr. 40 min. being added together, make 58 degr. which being taken from 180 degr. there will remain 122 deg. the quantity of the obtuſe Angle DCB.

18. In all Triangles whatſoever, the Sides are in proportion one to the other as the Sines of the Angles oppoſite to thoſe Sides. So in the Triangle CDB, the Sine of the Angle at D, is in proportion to the Side CB, which is oppoſite to it, as the Sine of the Angle at B, is to the Side CD, or the Angle at C, to the Side DB.

Theſe things being premiſed, I come now to the Solution of *Plain Triangles* both *Right* and *Oblique* angled.

I. *Of Right angled Plain Triangles.*

THE Triangle which I ſhall make uſe of in the ſeveral Caſes belonging to a *Right-angled Plain Triangle*, ſhall be that *Fig. I.* noted with CAB, In which

		parts	
A B the Baſe,	contains	180	
C A the Perpendicular,		135	
C B the Hypotenuſe,		225	
And		deg.	m.
A the Right Angle,	contains	90	00
C the Angle at the Per.		53	07
B the Angle at the Baſe,		36	53

CASE I.

The Baſe B A 180, *and the Perpendicular C A* 135, *being given, to find the Angles B and C.*

The Proportion is,

As the Logarithm of A B
Is to the Logarithm of A C,
So is the Radius,
To the Tangent of B.

Extend the Compaſſes from 180 the Baſe, to 135 the Perpendicular, upon the Line of Numbers, the ſame extent will reach, the ſame way, from the Radius (or Tangent of 45 deg.) to the Tangent of 36 deg. 53 min. the quantity of the Angle at B.

CASE II.

The Hypotenuſe C B 225, and the Baſe A B 180, being given, to find the Angles B and C.

The Proportion is,

As the Logarithme of C B,
Is to the Radius;

So is the Logarith. of the Side AB,
To the Sine of C.

Extend the Compaſſes from 225 the Hypotenuſe, to the Radius (or Sine of 90 degr.) the ſame extent will reach, the ſame way, from 180 the Baſe, to 53 deg. 7 min. the quantity of the Angle at C.

Or,

The diſtance between 225 and 180, will reach from the Sine of 90, to the Sine of 53 degr. 7 min. as before.

CASE III.

The Baſe AB 180, *the Angle C* 53 *degr.* 7 *min. and the Angle B* 36 *deg.* 53 *min. being given, to find the Perpendicular CA.*

The Proportion is,

As the Sine of the Angle at C,
Is to the Logar. of AB,
So is the Sine of the Angle B,
To the Logar of CA.

Or,

As the Radius,
Is to the Logar. of AB,
So is the Tangent of B,
To the Logar. of C A.

Extend the Compaſſes from the Sine of 53 deg. 7 min. the Angle at C, to 180 the Baſe, the ſame extent will reach from the Sine of 36 degr. 53 min. to 135 the Perpendicular C A.

Or,

Extend the Compaſſes from the Tangent of 45 deg. to 180 the Baſe, the ſame extent will reach, the ſame way, from the Tangent of 36 deg. 53 min. to 135 the Perpendicular, as before.

CASE IV.

The Hypotenuſe CB 225, the Angle C 53 deg. 7 min. and the Angle at B 36 deg. 53 min. given, to find the Baſe B A, and the Perpendicular C A.

The

The Proportion is,

As the Radius,
Is to the Logar. of CB,
So is the Sine of C,
To the Logar. of AB.
And the Sine of B,
To the Logar. of CA.

Extend the Compaſſes from the Sine of 90, to 225 the Hypotenuſe, the ſame extent will reach from the Sine of 53 deg. 7 min. the Angle at C, to 180 the Baſe AB —— And likewiſe, the ſame extent will reach from the Sine of 36 deg. 53 min. to 135, the Perpendicular CA.

CASE V.

The Hypotenuſe CB 225, and the Baſe AB 180, being given, to find the Perpendicular CA,

The Proportion is,

1. *Operation.*

As the Logar. of C B,
Is to the Radius ;
So is the Logar. of A B,
To the Sine of C.

2. *Operation.*

As the Radius,
Is to the Logarithm of C B,
So is the Sine of B (the Complement of C)
To the Logar. of C A.

Extend the Compaſſes from 225 the Hypotenuſe, to the Sine of 90, the ſame extent will reach from 180 the Baſe, to the Sine of 53 degr. 7 min. the Angle at C.

Again,

Extend the Compaſſes from the Sine of 90, to 225 the Hypotenuſe, the ſame extent will reach from the Sine of 36 degrees 53 minutes, the

Angle at B, to 135 the Perpendicular C A.

CASE VI.

The Baſe AB 180, *the Angle C* 53 *degr.* 7 *min. and the Angle B* 36 *degr.* 53 *min. being given, to find the Hypotenuſe C B.*

The Proportion is,

As the Sine of C,
Is to the Logar. of A B;
So is the Radius,
To the Logar. of C B.

Extend the Compaſſes from the Sine of 53 degr. 7 min. to 180 the Baſe, the ſame extent will reach from the Sine of 90, the Angle at A, to 225 the Hypotenuſe CB.

CASE VII.

The Baſe AB 180, *and the Perpendicular C A* 135, *being given, to find the Hypotenuſe C A.*

The Proportion is,

1. *Operation.*

As the Logar. of AB,
Is to the Logar. of CA;
So is the Radius,
To the Tangent of B.

2. *Operation.*

As the Sine of B,
Is to the Logarithm of CA,
So is the Radius,
To the Logar. of CB.

Extend the Compaſſes from 180 the Baſe, to 135 the Perpendicular, the ſame extent will reach from the Tangent of 45 degrees (or Radius) to the Tangent of 36 degr. 53 min. the Angle at B.

Again,

Extend the Compaſſes from the Sine of 36 deg. 53 min. the Angle at B, to 135 the Perpendicular CA; the ſame extent will reach from the

Sine of 90 degrees, to 225 the Hypotenuſe C B.

Theſe are the Seven Caſes of Right angled Plain Triangles, *I come now to the ſolution of the Five Caſes of* Oblique-angled Plain Triangles.

II. *Of Oblique-angled Plain Triangles.*

THe Triangle which I ſhall make uſe of in the Solution of the ſeveral Caſes appertaining to an *Oblique-angled Plain Triangle*, ſhall be that *Fig. II.* noted with C D B: In which,

		parts
D B the Baſe,	contain	335
C B the longer Side,		271
D C the ſhorter Side,		100

And

		deg.	*m.*
C, the Obtuſe Angle,	contains	122	00
D, the 2 Acute Angles,		43	20
B,		14	40

In the Solution of this *Oblique-angled Triangle*, I call the longeſt Side DB the Baſe, and the other two, the two Sides, without any other diſtinction, as of *Hypotenuſe* or *Perendicular*; for in theſe Triangles (taken entirely of themſelves) there is no ſuch diſtinction properly to be made.

CASE I.

Two Sides, as the Baſe DB 335, and the Side CB 271, and the Angle D 43 deg. 20 min. oppoſite to CB, to find the Angle at C, oppoſite to the Baſe DB.

Extend the Compaſſes from 271, the Side BC, (upon the Line of Numbers) to the Sine of 43 deg. 20 min. the Angle at D, the ſame extent will reach from 335 the Baſe DB (upon the Line of Numbers) to the Sine of 58 deg. the Complement of the Obtuſe Angle at C, to 180 deg. which is 122 deg.

CASE II.

The Baſe D B 335, *and the Side D C* 100, *with the Angle D*, 43 *degr.* 20 *min. contained between them, to find either of the other Angles at B and C.*

IN the ſolution of this Problem, you muſt firſt get the Sum and the Difference of the two given Sides.

Alſo you muſt get the Half ſum of the two unknown Angles, in this manner,

The two given ſides are	DB	335
	DC	100
	Their Sum	435
	Their difference	235

The

	deg.	*m.*
The given Angle is	43	20
Which ſubtracted from 180 deg. leaves	136	40

This 136 deg. 40 m. is the quantity of both the Angles at C and B,
The half whereof is 68 20

Being thus prepared,
The Proportion is,

As the Logar. of the ſum of the two Sides given, CD and CB,
Is to the difference of thoſe Sides;
So is the Tangent of half the Sum of the two unknown Angles C and B,
To the Tangent of half their difference.

Extend the Compaſſes from 435 (the Sum of the two Sides DB and DC) to 235 (the difference of the ſame two Sides) that extent will reach from the Tangent of 68 deg. 20 min. (the half ſum of the two unknown Angles C and B) to the Tangent of 53 deg. 40 min.

This 53 deg. 40 min. being added to 68 deg. 20 m. (the half ſum of the Angles B and C) gives 122 deg. for the greater Angle C, and being ſubtracted from 68 deg. 20 min. leaves 14 deg. 40 min. for the leſſer Angle at B.

CASE III.

The three Sides DB 335, CB 271, and DC 100, being given, to find any of the Angles, as B.

BEfore you can reſolve this Problem, you muſt obtain the Sum and Difference of the two Sides containing the Obtuſe Angle, thus,

$$\text{The Side}\begin{Bmatrix}CB\\CD\end{Bmatrix}\text{is}\begin{Bmatrix}271\\100\end{Bmatrix}$$

Their Sum 371

Their difference 171
The Baſe is 335
Which

Which being known,

The Proportion is,

As the Logarithm of the greater Side DB,

Is to the Sum of the other two Sides, DC and CB;

So is the difference of the two Sides,

To a fourth Sum,

Which fourth Sum, being taken from the Baſe, will leave another number, the half whereof will be the place in the Baſe where a Perpendicular let fall from the Obtuſe Angle, would fall upon the Baſe: and ſo the Oblique Triangle is reduced into two Right-angled, and may be reſolved by the Precepts of Right-angled Triangles,

Extend the Compaſſes from 335 (the greateſt Side, to 371 (the ſum of the other two ſides CD and CB) the ſame extent will reach from

171 (the difference of the ſame ſides) to 189,4.

This number 189,4 (being ſubſtracted from the Baſe DB 335) there will remain 145,6, the half whereof is 72,8, which is the length of the Baſe from D to E; in which point a Perpendicular let fall from the Obtuſe angle C, will fall: and ſo the Obtuſe angled Triangle CDB, is reduced into two Right-angled Triangles C E D and C E B, and any of the parts of either of them may be found by the precepts for the reſolving of Right-angled Triangles.

CASE IV.

The three Angles C 122 *degr. D* 43 *degr.* 20 *min. and B* 14 *degr.* 40 *min. being given, to find any of the Sides, as B C.*

IN this Caſe, where the three Angles are given, and a Side required,

red, no abſolute proportion can be preſcribed; for the Sides cannot abſolutely be found, but their proportions one to another may be obtained; for that the three Angles of one Triangle may be equal to the three Angles of another Triangle, although their Sides be altogether unequal.

CASE V.

The two Sides D C 100, and C B 271, with the Angle at C 122 degr. being given, to find the Baſe D B.

YOu muſt firſt find (by the foregoing Caſes the two Angles at D and B: Making choice of one of the Sides, as CD, to work your Proportion by: Then,

The Proportion will be,

As the Sine of B,
Is to the Sine's Compl. of C;

So is the Logarithm of C D,
To the Logarithm of D B.

Extend the Compaſſes from the Sine of 14 degr. 40 min. the Angle at B, to the Sine of 58 degr. the Complement of the Angle at C, to 180 deg. the ſame Extent will reach (upon the Line of Numbers) from 100 the Side C D, to 335 the Side D B.

And theſe are all the Caſes that can ariſe in the Solution of any Oblique-angled Triangle, I will proceed to the Solution of Spherical Triangles.

SPHERICAL TRIGONOMETRY.

I. *Of Right-angled Spherical Triangles.*

THe Right-angled Spherical Triangle, which I ſhall make uſe of in the following XVI. Caſes, ſhall be the Triangle C B A, *Fig. IV.* right-angled at A; the quantity of each Side and Angle being as followeth; *viz.*

		d.	m	Comp.	
The Hypotenuſe C B		30	00	60	00
The Baſe B A		27	54	62	06
The Perpendicular C A		11	30	78	30
Angle at the	Baſe B	23	30	66	30
	Perpendicular C	69	36	20	24

CASE

CASE I.

The Hypotenuſe and Angle at the Baſe given, to find the Perpendicular.

The Proportion is,

As Radius 90,
Is to Sine B C 30,
So is Sine B 23 deg. 30 min.
To the Sine C A 11 deg. 30 min.

Extend the Compaſſes from the Sine of 90, to the Sine of 30, the ſame extent will reach from the Sine of 23 deg. 30 min. to the Sine of 11 deg. 30 min. the Perpendicular.

CASE II.

The Hypotenuſe and Perpendicular given, to find the Baſe.

The Proportion is,

As the Co-ſine C A 78 deg. 30 m.
Is to Radius 90,

So is Co-ſine CB 60 ,
To Co-ſine B A 62 deg. 6 min.

Extend the Compaſſes from the Sine of 78 deg. 30 min. to 90, the ſame extent will reach from the Sine of 60 deg. to the Sine of 62 deg. 6 min. the Baſe.

CASE III.

The Angles at the Baſe and Perpendicular given, to find the Perpendicular?

The Proportion is,

As Sine C 69 deg. 36 m.
Is to Co-ſine B 66 deg. 30 m.
So is Radius 90 deg.
To Co-ſine C A 79 deg. 30 m.

Extend the Compaſſes from the ſine of 69 deg. 36 min. to the ſine of 90 , the ſame extent will reach from the ſine of 66 deg. 30 min. to the ſine of 79 deg. 30 min. the complement of the Perpendicular.

CASE

CASE IV.

The Baſe, and Angle at the Baſe given, to find the Perpendicular.

The Proportion is,

As Radius 90 deg.
Is to ſine B A 27 deg. 54 min.
So is tangent B 23 deg. 30 min.
To tangent C A 11 deg. 30 min.

Extend the Compaſſes from the ſine of 90, to the ſine of 27 deg. 54 min. the ſame extent will reach from the tangent of 23 deg. 30 m. to the tangent of 11 deg. 30 m. the Perpendicular.

CASE V.

The Perpendicular and Angle at the Baſe given, to find the Baſe.

The Proportion is,

As tangent B 23 deg. 30 min.
Is to tangent C A 11 deg. 30 m.

So is Radius 90 degr.
  To ſine B A 27 deg. 54 min.

Extend the Compaſſes from the tangent of 23 deg. 30 min. to the tangent of 11 deg. 30 min. the ſame extent will reach from the ſine of 90 deg. to the ſine of 27 deg. 54 m. for the Baſe.

CASE VI.

The Hypothenuſe and Angle at the Perpendicular given, to find the Perpendicular.

The Proportion is,

As Radius 90 deg.
  To co-ſine C 20 d. 24 m.
So is tangent B C 30 deg.
  To tangent C A 11 deg. 30 m.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 20 deg. 24 min. the ſame extent will reach from the tangent of 30 degr. to the tangent of 11 deg. 30 min, the perpendicular.

CASE VII.

The Angles at the Baſe and Perpendicular given, to find the Hypothenuſe.

The Proportion is,

As tangent C 69 deg. 36 min.
Is to the co-tangent B 66 d. 30 m.
So is Radius 90 d.
To co-ſine C B 60 d.

Extend the Compaſſes from the tangent of 69 d. 36 m. to the tangent of 66 d. 30 m. the ſame extent will reach from the ſine of 90 d. to the ſine of 60 d. for the complement of the Hypothenuſe.

CASE VIII.

The Baſe and Perpendicular given, to find the Hypothenuſe.

The Proportion is,

As Radius 20 d.
Is to co-ſine B A 62 d. 6 m.
So is coſine C A 79 d. 30 m.
To the co-ſine C B 60 d.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 62 deg. 6 min. the ſame extent will reach from the ſine of 79 d. 30 m. to the ſine of 60 d. for the complement of the Hypothenuſe.

CASE IX:

The Perperdicular and Angle at the Baſe given, to find the Hypothenuſe

The Proportion is,

As ſine B 23 d. 30 m.
Is to Radius 90 d.
So is ſine C A 11 d. 30 m.
To the ſine C B 30 d.

Extend the Compaſſes from the ſine of 23 d, 30 m. to 90 d. the ſame ex-

extent will reach from the ſine of 11 d. 30 m. to the ſine of 30 d. for the Hypotenuſe.

CASE X.

The Baſe, and the Angle at the Baſe, given, to find the Hypotenuſe.

The Proportion is,

As Radius 90 d.
Is to co-ſine B 66 d. 30 m.
So is co-tangent B A 62 d. 6 m.
To the co-tangent B C 60 d.

Extend the Compaſſes from the ſine of 90, to the ſine of 66 deg. 30 m. the ſame extent will reach from the tangent of 62 d. 6 m. to the tangent of 60 d. for the complement of the Hypotenuſe.

CASE XI.

The Perpendicular and Angle at the Perpendicular given, to find the Angle at the Baſe.

The Proportion is,

As Radius 90,
Is to Sine C 69 d. 36 m.
So is Co-ſine C A 79 d. 30 m.
To Co-ſine B 66 d. 30 m.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 69 d. 36 m. the ſame will reach from the ſine of 79 deg. 30 min. to the ſine of 66 deg. 30 min. for the Complement of the Angle at the Baſe.

CASE XII.

The Perpendicular and Angle at the Baſe given, to find the Angle at the Perpendicular.

The Proportion is,

As the Co-ſine C A 79 d. 30 m.
Is to Co-ſine B 66 d. 30 m.
So is Radius 90 degr.
To Sine C 69 d. 36 m.

Extend the Compaſſes from the ſine of 79 deg. 30 min. to the ſine of 66 deg. 30 min. the ſame extent will reach from the ſine of 90 deg. to the ſine of 69 deg. 36 min. for the angle at the Perpendicular.

CASE XIII.

The Hypotenuſe and Perpendicular given, to find the Angle at the Baſe.

The Proportion is,

As the Sine B C 30 deg,
Is to Radius 90 deg.
So is Sine C A 11 d. 30 m.
To Sine B 23 d. 30 m.

Extend the Compaſſes from the ſine of 30 deg. to the ſine of 90 degr. the ſame will reach from the ſine of 11 deg. 30 m. to the ſine of 23 deg. 30 min. for the angle at the Baſe.

CASE XIV.

The Baſe and Perpendicular given, to find the Angle at the Baſe,

The Proportion is,

As Sine B A 27 d. 54 m.
Is to Radius 90 d.
So is Tangent CA 11 d. 30 m.
To Tangent B 23 d. 30 m.

Extend the Compaſſes from the ſine of 27 deg. 54 min. to the ſine of 90 degrees, the ſame extent will reach from the tangent of 11 degr. 30 min. to the tangent of 23 degr. 30 min. for the angle at the Baſe.

CASE XV.

The Hypotenuſe and Perpendicular given, to find the Angle at the Perpendicular.

The Proportion is,

As the Tangent B C 30 d.
To the Tangent C A 11 d. 30 m.
So is Radius 90 degr.
To the Co-ſine C 20 d. 24 m.

Extend the Compaſſes from the tangent of 30 degr. to the tangent of 11 deg. 30 min. the ſame extent will reach from the ſine of 90 deg. to the ſine of 20 degr. 24 min. for the complement of the angle at the Perpendicular.

CASE XVI.

The Hypotenuſe and Angle at the Perpendicular given, to find the Angle at the Baſe.

The Proportion is,

As Radius 90 d.
Is to Co-ſine C B 60 d.

So is Tangent C 69 d. 36 m.
To Co-tangent B 66 d. 30 m.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 60 deg. the ſame extent will reach from the tangent of 69 degr. 36 min. to the Tangent of 66 degr. 30 min. for the Complement of the Angle at the Baſe.

Theſe 16 Caſes are all that can be propoſed in a Right-angled Spherical Triangle. There are 12 other Caſes which belong to Oblique-angled Spherical Triangles, which we now come to reſolve.

II. *Of Oblique-angled Spherical Triangles.*

THe Triangle which I ſhall make uſe of in the ſolution of the 12 Caſes of Oblique-angled Spherical Triangles, ſhall be *Fig. IV.* Z S P, whoſe Sides and Angles are as followeth; *viz.*

		d.	m.	Comp.	
The Side	S P	70	00	20	00
	Z P	38	30	51	30
	Z S	40	00	50	00
The Angle	Z	130	8	49	52
	S	30	24	59	36
	P	31	32	58	28

The Perpendicular Z R.

The Segment	S R	35	54	54	06
	P R	34	06	55	54

CASE I.

Two Sides ZP *and* ZS, *and the Angle* S, *oppoſite to* Z P, *given, to find the Angle* P *oppoſite to the other ſide* Z S.

The Proportion is,

As Sine ZP 38 d. 30 m.
Is to the Sine S 30 d. 24 m.
So is Sine SZ 40 d. 0 m.
To Sine P 31 d. 32 m.

Extend the Compaſſes from the Sine of ZP 38 deg. 30 min. to the Sine of S 30 deg. 24 min. the ſame extent will reach from the Sine of SZ 40 deg. to the Sine of 31 deg. 32 min. the Angle at P.

CASE II.

Two Angles Z *and* P, *with the Side* SP, *oppoſite to* Z *given, to find the Side* SZ, *oppoſite to the Angle* P.

The Proportion is,

As Sine Z 130 d. 8 m.
Is to Sine S P 70 d.
So is Sine P 31 d. 32 m.
To Sine S Z 40 d.

Extend the Compaſſes from the Sine of 49 deg. 52 min. (the Complement of 130 deg. 8 min. to 180 deg.) to 70 deg. the ſame extent will reach from 31 deg. 32 min. to the Sine of 40 degr. for S Z.

In the reſolving of the following Caſes of Oblique Spherical Triangles, it will be neceſſary to reduce the Oblique Triangle into two Right-angled Triangles; which muſt be effected by letting fall of a Perpendicular from an Angle upon a Side oppoſite thereunto; and for the letting fall of this Perpendicular, obſerve theſe few following Directions.

Directions for the letting fall of the Perpendicular in any Oblique Spherical Triangle.

1. The Perpendicular muſt fall from an unknown Angle upon a Side oppoſite thereto.
2. By the Perpendicular ſo let fall, the Oblique Triangle is reduced into two Right-angled Triangles.
3. The Perpendicular falls ſometimes within the Triangle, ſometimes without; *viz.*

When the Angles at the ends of that Side upon which the Perpendicular is to fall, be	both Acute both Obtuſe	The Perpend. falls	Within
	One Acute and the other Obtuſe,		Without.

That Side muſt alwayes be the Baſe, upon which the Perpendicular is to fall; and muſt be extended (if need require.)

CASE III.

Two Sides SZ *and* SP, *with the Angle* S *included between them, given, to find the oppoſite Angle* P.

IN this Caſe the Baſe is that given ſide which is adjacent to the Angle ſought, namely S P.

The Proportion is,

1, *Operation.*

As Radius 90 d.
Is to Co-ſine 59 d. 36 m.
So is Tangent ZS 40 d.
To Tangent SR 35 d. 54 m.

2. *Operation.*

As Sine SR 35 d. 54 m.
To Sine RP 34 d. 6 m:
So is Tangent S 30 d. 24 m.
To Tangent P 31 d. 32 m.

I.

Extend the Compaſſes from Radius 90 deg. to 59 deg. 36 min. the ſame extent will reach from Tangent 40 deg. to the Tangent of 35 deg. 54 min. for S R.

II.

Extend the Compaſſes from the Sine of 35 deg. 54 min. to the Sine of 34 deg. 6 min. the ſame extent will reach from the Tangent of S 30 deg. 24 min. to the Tangent of 31 deg. 32 min. for the Angle at P.

CASE IV.

Two Sides Z P *and* P S, *with the Angle* P *included between them, given, to find the Side* Z S, *oppoſite to the given Angle* P.

IN this Caſe the Baſe is one of the given Sides.

The Proportion is,

1. *Operation.*

As Radius 90 d.
Is to Coſine P 58 d. 28 m.

So is Tangent Z P 38 d. 30 m.
To Tangent P R 34 d. 6 m.

2. Operatirn.

As Co-ſine P R 55 d. 54 m.
Is to Co-ſine S R 54 d. 6 m.
So is Co-ſine Z P 51 d. 30 m.
To Co-ſine Z S 50 d.

I.

Extend the Compaſſes from the Sine of 90 deg. to the Co-ſine of P 58 deg. 28 min. the ſame extent will reach from the Tangent of Z P 38 deg. 30 min. to the Tangent of 34 deg. 6 min. for P R.

II.

Extend the Compaſſes from the Co-ſine of P R 55 deg. 54 min. to the Co-ſine of S R 54 deg. 6 min. the ſame extent will reach from the Co-ſine of Z P 51 deg. 30 min. to the Co-ſine of 50 deg. for Z S.

CASE V.

Two Sides ZP *and* SP, *with the Angle* P *contained between them, given, to find the Angle* S, *oppoſite to the Angle* P.

The Proportion is,

1. *Operation.*

As Radius 90 d.
Is to Co-ſine P 58 d. 28 m.
So is Tangent ZP 38 d. 30 m.
To Tangent RP 34 d. 6 m.

2. *Operation.*

As Sine PR 34 d. 6 m.
Is to Sine SR 35 d. 54 m.
So is Tangent P 31 d. 32 m.
To Tangent S 30 d. 24 m.

I.

Extend the Compaſſes from Radius 90 deg. to the Co-ſine of P 31 deg. 32 min. the ſame extent will reach from the Tangent of ZP 38 deg. 30 min. to the Tangent of 30 deg. 24 min. for the Angle at S.

II.

Extend the Compaſſes from the Sine of P R 34 d. 6 m. to the Sine of S R 35 d. 54 m. the ſame extent will reach from the Tangent P 31 deg. 32 m. to the Tangent of 30 deg. 24 m. for the Angle at *S.*

CASE VI.

Two Sides Z P *and* Z S, *with the Angle* P *oppoſite to* S Z, *given, to find the Angle* Z, *contained between the two given Sides.*

IN this Caſe the Baſe is alwayes the Side unknown.

The Proportion is,

1. *Operation.*

As Radius 90 d.

Is to Co-ſine Z P 38 d. 30 m.

So is Tangent P 31 d. 32 m.
To Tangent RZP 25 d. 38 m.

2. *Operation.*

As Tangent ZS 40 d.
Is to Tangent ZP 38 d. 30 m.
So is Co-ſine RZP 64 d. 22 m.
To Co-ſine SZR 58 d. 44 m.

I.

Extend the Compaſſes from Radius 90 deg. to the Co-ſine of ZP 51 deg. 30 min. the ſame extent will reach from the Tangent of P 31 deg. 32 min. to the Tangent of 25 deg. 38 min. for the Angle RZP.

II.

Extend the Compaſſes from the Tangent of ZS 40 deg. to the Tangent of ZP 38 deg. 30 min. the ſame extent will reach from the Co-ſine of RZP 64 deg. 22 min. to the Co-ſine of 31 deg. 16 min.

CASE

CASE VII.

Two ſides Z S and Z P, with the angle S oppoſite to Z P given, to find the ſide S P adjacent to the given angle S.

The Proportion is,

1. Operation.

As Radius 90 d.
Is to co-ſine S 59 d. 36 m.
So is tangent Z S 40 d.
To tangent S R 35 d. 54 m.

2. Operation.

As co-ſine S Z 50 d.
Is to co-ſine Z P 51 d. 30 m.
So is co-ſine S R 54 d. 6 m.
To co-ſine P R 55 d. 54 m.

I.

Extend the Compaſſes from Radius 90 deg. to the co-ſine of S 59 d. 36 m. the ſame extent will reach from the tangent of Z S 40 deg. to the tangent of S R 35 deg. 54 min.

II.

II.

Extend the Compaſſes from the co-ſine of SZ 50 deg. to the coſine of ZP 51 deg. 30 min. the ſame extent will reach from the co-ſine of SR 54 deg. 6 min. to the co-ſine of PR 55 deg. 54 min.

CASE VIII.

Two angles S and Z, with the ſide SZ included between them given, to find the angle P oppoſite to the given ſide SZ.

IN this Caſe the Baſe may be either of the unknown ſides.

The Proportion is,

1. *Operation.*

As Radius 90 d.
Is to co-ſine *S*Z 50 d.
So is tangent *S* 30 d. 24 m.
To co-tangent RZ*S*.

2. *Operation.*

As ſine RZS
To ſine RZP;
So is co-ſine S 59 d. 36 m.
To co-ſine P 58 d. 28 m.

I.

Extend the Compaſſes from Radius 90 deg. to the co-ſine of SZ 50 deg. the ſame extent will reach from the tangent of S 30 degrees 24 minutes, to the co-tangent of RZS.

II.

Extend the Compaſſes from the ſine of RZS to the ſine of RZP, the ſame extent will reach from the co-ſine of S 59 degrees 36 minutes, to the co-ſine of P 58 degrees 28 minutes.

CASE IX.

Two angles Z and P, with the ſide ZP between them, given, to find the ſide ZS oppoſite to the given angle at P.

IN this Caſe the Baſe is the ſide neither given nor ſought as SP.

The Proportion is,

1 *Operation.*

As Radius 90 d.
Is to co-ſine ZP 51 d. 30 m.
So is tangent P 31 d. 32 m.
To co-tangeut RZP.

2. *Operation.*

As co-ſine RZS
Is to co-ſine RZP,
So is tangent ZP
To tangent ZS

I.

Extend the Compaſſes from Radius 90 degr. to the co-ſine of ZP 51

51 degr. 30 min. the ſame extent will reach from the tangent of *P* 31 degr. 32 min. to the co-tangent of RZ*P*.

II.

Extend the Compaſſes from the co-ſine of RZ*S*, to the co-ſine of RZP, the ſame extent will reach from the tangent of ZP 38 deg. 30 min. to the tangent of ZS 40 degrees.

CASE X.

Two angles S *and* P, *with a ſide oppoſite to one of them* SZ, *given, to find the other angle* Z.

IN this Caſe the Baſe is the ſide oppoſite to the angle ſought.

The Proportion is;

1. *Operation.*

As Radius 90 d.
Is to co-ſine ZS 50 d.

So is tangent S 30 d. 24 m.
To co-tangent SZR.

2. *Operation.*

As co-ſine S 59 d. 36 m.
Is to co-ſine P 58 d. 28 m.
So is ſine SZR,
To ſine RZP.

I.

Extend the Compaſſes from Radius 90 degr. to the co-ſine of ZS 50 degr. the ſame extent will reach from the tangent of S 30 degr. 24 min. to the co-tangent of SZR.

II.

Extend the Compaſſes from the co-ſine of S 59 degr. 36 min. to the co-ſine of P 58 d. 28 m. the ſame extent will reach from the ſine of SZR to the ſine of RZ*P*.

CASE XI.

The three ſides S Z, P Z, and S P, given, to find an angle, viz *the angle at Z.*

IN this Caſe the ſide oppoſite to the inquired angle is the Baſe.

Before the Triangle can be reſolved, you muſt

Firſt, Adde the three ſides together, and note the ſum of them.

Secondly, Take the half thereof, which call the half ſum.

Thirdly, From the half ſum, ſubſtract the Baſe, and note the difference, as you ſee here done.

The Side	SZ	40	00
	ZP	38	30
	SP	70	00
The Sum		148	30
The half Sum		74	15
From which ſubſtract the Baſe 70 deg there remains the difference		4	15

This preparation being made, the proportion will be,

1 *Operation.*

As Radius 90 d.
Is to ſine ZS 40 d.
So is the ſine of Z*P* 38 d. 30 m.
To a fourth ſine, *viz.* 23 d. 35 m.

2 *Operation.*

As the ſine of 23 d. 35 m.
Is to the ſine of the half ſum 74 d. 15 m.
So is the ſine of the difference 4 d. 15 m.
To a ſeventh ſine, *viz.* 10 d. 17 m.

I.

Extend the Compaſſes from Radius 90 deg. to the ſine of Z S 40 deg. the ſame extent will reach from the ſine of Z P 38 deg. 30 min. to a fourth ſine, *viz.* 23 deg. 35 min.

II.

Extend the Compaſſes from the ſine of 23 deg. 35 min. to the ſine of the half ſum 74 degr. 15 min. the ſame extent will reach from the ſine of the difference 4 deg. 15 min. to a ſeventh ſine, *viz.* 10 deg. 17 min.

Divide the ſpace upon the Line of Sines betwen 10 deg. 17 min. and 90 degr. into two equal parts, and the Compaſs point ſhall reſt upon 24 deg. 56 min. whoſe Complement is 65 deg. 4 min. and that doubled makes 130 deg. 8 min. for the angle at Z.

CHAP. XII.

The three angles Z, S, and P, given, to find a ſide.

THis is but the converſe of the former Caſe, and may be reſolved in the ſame manner, if for either of the angles next to the ſide required, you take its complement to 180 deg. thoſe angles will be turned into ſides, and the ſides into angles; and then may the triangle be reſolved as in the preceding Caſe.

The USE of the PROPORTIONAL LINES IN *ASTRONOMY.*

CHAP. V.

Argument.

I Shall not in this place go about to give you any Deſcription of the Circles of the Sphere or Globe, ſuppoſing my Reader to be acquainted with them already; and in reſpect I have ſufficiently treated of them elſewhere, as in my *Uſes of the Globes*, and alſo in my *Geometrical Exerciſes*; which Book will explain and make eaſie ſome things which in this Tractate may be omitted, or at leaſt, for brevity, lightly paſſed over.

Probl. I.

The diſtance of the Sun *from the neareſt* Æquinoctial Point (*either* Aries *or* Libra) 59 *deg. given, to find his* Declination.

The Proportion is,

As the Radius 90 deg.
Is to the Sine of the Sun's greateſt Declination 23 deg. 30 m.
So is the Sine of the Sun's diſtance from the next Æquinoctial Point *Libra* 59 degr.
To the ſine of the Sun's preſent Declination 20 degr.

Extend the Compaſſes from the ſine of 90, to the ſine of 23 deg. 30 min. (the *S*uns greateſt Declination) the ſame extent will reach from 59 deg. (the Suns diſtance from *Libra*,

Libra, to the ſine of 20 deg. the Suns preſent Declination.

The like Declination *the* Sun *hath when he is in* 29 *degr. of* Taurus, *in* 1 *degr. of* Leo, *or* 29 *degr. of* Scorpio, *every of which Points are diſtant from one of the* Æquinoctial Points Aries *or* Libra 59 *deg.*

Probl. II.

The Latitude *of the Place*, 51 *deg.* 30 *min. and the* Declination *of the* Sun 20 *deg. being given, to find the* Aſcenſional Difference.

The Proportion is,

As the co-tangent of the Latitude 38 deg. 30 min.
Is to the tangent of the Suns Declination 20 degr.
So is the Radius 90 deg.
To the ſine of the Aſcenſional Difference 27 deg. 14 min.

Extend the Compaſſes from the tangent of 38 deg. 30 min. the complement of the Latitude, to 20 deg. (the Suns Declination) the ſame extent will reach, the ſame way, from the ſine of 90 deg. to the ſine of 27 deg. 14 min. the Aſcenſional difference; which is the quantity of time that the Sun riſes or ſets before or after Six of the Clock.

So theſe 27 degr. 14 min. being turned into Time (by allowing 15 deg. for one hour, and one degree for 4 minutes of Time) is 1 hour and 49 min. and ſo much doth the Sun riſe or ſet before or after the hour of Six, according to the time or ſeaſon of the year; for if the Sun hath *North Declination*, then he *riſeth before ſix* and *ſets after:* but if the Sun have *South Declination*, then doth he *riſe after*, and *ſets before Six.*

This Aſcenſional Difference *being added to Six hours, will give you the* Se-

Semidiurnal Arch *or Half-length of the Day*; *and being taken from Six hours, will leave the* Seminocturnal Arch, *or Half-length of the Night.*

Probl. III.

The Latitude *of the Place* 51 *deg.* 30 *min. and the* Declination *of the Sun*, 20 *deg. being given, to find his* Amplitude.

The Proportion is,

As the co-ſine of the Latitude 38 deg. 30 min.

Is to the Radius 90 deg.

So is the ſine of the Sun's Declination 20 degr.

To the ſine of the Amplitude from the Eaſt or Weſt points of the Horizon 33 degr. 20 min.

Extend the Compaſſes from the ſine of 38 deg. 30 min (the Complement of the Latitude) to the ſine of 90 deg. the ſame extent will reach from the ſine of 20 deg. (the Suns Declination) to 33 deg. 20 min. (the Amplitude, or) the diſtance that the Sun riſes or ſets from the true Eaſt or Weſt Points, towards either the North or South.

Probl. IV.

The Latitude *of the Place*, 51 *deg*, 30 *min. and the* Declination *of the Sun* 20 *deg. being given, to find the* Angle of the Sun's Poſition *at the time of his riſing.*

The Proportion is,

As the co-ſine of the Declination 70 degr.
Is to the Radius 90 degr.
So is the ſine of the latitude 51 degr. 30 min.
To the ſine of the Angle of the Suns Poſition at the time of his riſing.

Exrend the Compaſſes from the ſine of 70 deg. (the complement of the Suns Declination) to the ſine of 90; the ſame extent will reach from the ſine of 51 deg. 30 min. the latitude) to the ſine of 56 deg. 29 min. (the angle of the Sun's poſition at the time of his riſing.)

Probl. V.

The Sun's Declination 20 *deg. and his* Amplitude 33 *deg.* 20 *min. from the* Eaſt *or* Weſt *part of the* Horizon, *being given, to find the* Latitude.

The Proportion is,

As the ſine of the Amplitude from the Eaſt or Weſt 33 deg. 20 min.

Is to the Radius 90 deg.

So is the ſine of the Declination 20 deg.

To the co-ſine of the Latitude 38 degr. 30 min.

Extend the Compaſſes from the ſine of 33 deg. 20 min (the Sun's Amplitude from the Eaſt or Weſt) to the ſine of 90 deg. the ſame extent will reach from the ſine of 20 deg. (the Sun's Declination) to the ſine of 38 deg. 30 min. (the complement of the Latitude, 51 deg. 30 min.)

Probl. VI.

The Sun's greateſt Declination 23 *deg.* 30 *min. with his Diſtance from the next* Æquinoctial Point (Aries *or* Libra, 59 *deg.*) *being given, to find his* Right Aſcenſion.

The Proportion is,

As the Radius 90 deg.

Is to the co-ſine of the greateſt Declination 66 deg. 30 min.

So is the tangent of the Sun's diſtance from the next Æquinoctial point *Libra* 59 deg.

To the tangent of the Right Aſcenſion 56 deg. 50 min.

Extend the Compaſſes from the

ſine of 90 deg. to the ſine of 66 deg. 30 min. (the complement of the Sun's greateſt Declination;) the ſame extent will reach from the tangent of 59 deg. (the Suns diſtance from the next Æquinoctial Point) to the tangent of 56 deg. 50 min. (the Suns Right Aſcenſion.)

Probl. VII.

The Latitude *of the Place 51 deg.* 30 *min. and the Suns* Declination 20 *deg. being given, to find at what hour the Sun will be upon the true* Eaſt *or* Weſt *Points.*

The Proportion is,

As the tangent of the Latitude 51 deg. 30 min.
Is to the tangent of the Suns Declination 20 degr.
So is the Radius 90 degr.
To the co-ſine of the Hour from Noon.

Extend the Compaſſes from the tangent of 51 deg. 30 min (the Latitude) to the tangent of 20 deg. (the Suns Declination) the ſame extent will reach from the ſine of 90 deg. to the ſine of 16 deg. 50 min. the complement of the time from Noon, that the Sun will be due Eaſt or Weſt.

Which converted into hours and minutes, will be 4 hours and about 53 min. So that the Sun, when he hath 20 degr. of Declination, will come to the Eaſt Point at 7 min. paſt 7 in the Morning, and will be due Weſt 53 min. after 4 in the Afternoon.

Probl. VIII.

Having the Latitude of the Place 51 *deg.* 30 *min. and the Suns* Declination 20 *deg. given, to find what* Altitude *the Sun ſhall have when he is upon the true* Eaſt *or* Weſt *Points.*

The Proportion is,

As the ſine of the Latitude 51 deg. 30 min.

Is to the Sine of the Declination 20 degr.

So is the Radius 90 degr.

To the Sine of the Suns Altitude being due Eaſt or Weſt 25 degr. 55 min.

Extend the Compaſſes from the ſine of 51 deg. 30 min. (the Latitude) to the ſine of 20 deg. (the Declination) the ſame extent will reach from the ſine of 90 deg. to the ſine 25 deg. 55 min. the Altitude that the Sun ſhall have when he is upon the Eaſt or Weſt Points.

Probl. IX.

The Latitude *of the Place* 51 *deg.* 30 *min. and the Suns* Declination 20 *deg. being given, to find what* Altitude *the Sun ſhall have at Six of the Clock.*

The Proportion is,

As the Radius 90 degr.

Is to the ſine of the Suns Declination 20 degr.

So is the ſine of the Latitude 51 deg. 30 min.

To the ſine of the Suns Altitude at Six, 15 deg. 30 min.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 20 deg. (the Suns Declination) the ſame extent will reach from the ſine of 51 deg. 30 min. (the Latitude) to the ſine of 15 deg. 30 min. (the Altitude that the Sun ſhall have at Six of the Clock.)

Probl. X.

The Latitude *of the Place* 51 *deg.* 30 *min. and the* Declination *of the Sun* 20 *deg. being given, to find what* Azimuth *the Sun ſhall have at Six a Clock.*

The Proportion is,

As the co-ſine of the Latitude 38 degr. 30 min.
Is to the Radius 90 deg.
So is the co-tangent of the Suns Declination 70 degr.
To the tangent of the Suns Azimuth counted from the North part of the Meridian 77 deg. 14 min.

Extend the Compaſſes from the ſine of 38 deg. 30 min (the complement of the Latitude) to the ſine of 90 deg. the ſame extent will reach from the tangent of 70 deg. (the complement of the Suns Declination) to the tangent of 77 deg. 14 min.) the Suns Azimuth counted from the North part of the Meridian) or 12 degr. 46 min. the Azimuth from the Eaſt or Weſt, or 102 deg. 46 min. from the South,

Probl.

Probl. XI.

The Latitude *of the Place* 51 *deg.* 30 *min. the* Declination *of the Sun*, 20 *deg.* South, *and the Suns* Altitude 12 *deg. given*, *to find the Suns* Azimuth *either from the* Eaſt, North, *or* South *Points of the* Horizon.

TO reſolve this Problem, you muſt find the Complement of the Latitude, the Complement of the Altitude, and the Complement of the Declination, and add all three of them into one Snm, and take the half thereof; from which half ſum ſubſtract the Complement of the Suns Declination, and note the difference; as you ſee here done.

Complement of the	Latitude	38	30
	Altitude	78	00
	Declinat.	110	00
	Their Sum	226	30
	Half Sum	113	15
	Comp. Declinat. ſub.	110	00
	The Difference	3	15

Having found the Sum , the half Sum, and the Difference, you may work by the following

Proportion,

1. As the Radius 90 degr.
Is to the co-ſine of the Latitude, 38 deg. 30 min.
So is the co-ſine of the Altitude 78 degr.
To the ſine of a fourth number; which is 37 deg. 30 min.

2. As the ſine of the fourth number 37 deg. 30 min.

Is to the ſine of the half Sum 113 deg. 15 min.

So is the ſine of the Difference 3 deg. 15 min.

To another ſine, *viz.* 4 degr. 54 min.

Unto which ſeventh ſine, if you adde the ſine of 90 *degr. half that Sum ſhall be the ſine of an Arch; whoſe Complement being doubled is the Azimuth from the North.*

I.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 38 deg. 30 min. (the Complement of the Latitude) the ſame extent will reach from the ſine of 78 deg. (the Complement of the Altitude) to the ſine of 37 deg. 30 min.

II.

Extend the Compaſſes from the Sine of 37 deg. 30 min. the number laſt found, to the ſine of the half Sum 113 deg. 15 min. (or inſtead thereof, to 66 deg. 45 min. the Complement thereof to 180 deg.) the ſame extent will reach from the ſine of 3 degr. 15 min. (the Difference) to 4 deg. 54 min.

Divide the diſtance between 4 deg. 54 min. and 90 deg. into two equal parts, and the Compaſs point will reſt upon 16 deg. 48 min. the Complement whereof is 73 deg. 12 min. whoſe double 146 deg. 24 min. is the Suns Azimuth from the North part of the Meridian; and if you take that from 180 deg. there will remain 33 deg. 36 min. for the Suns Azimuth from the South.

Probl.

Probl. XII.

The Latitude *of the Place* 51 *deg.* 30 *min. the Suns* Declination 20 *deg. South, and the Suns* Altitude 12 *deg. given, to find the* Hour *of the Day.*

ADde the Complement of the Latitude, the Complement of the Declination, and the Complement of the Altitude, and take their Sum and half Sum, and from the half Sum ſubſtract the Complement of the Suns Altitude, and note the Difference.

Complement of the	Latitude	38	30
	Declination	70	00
	Altitude	78	00
	Sum	186	30
	Half Sum	93	15
	Difference	15	15

Being thus prepared,

The Proportions are,

(1.) As the Radius 90 degr.
Is to the co-ſine of the Latitude 38 deg. 30 min.
So is the co-ſine of the Altitude 78 degr.
To a fourth ſine, *viz.* 37 degr. 30 min.

(2.) As this fourth ſine of 37 deg. 30 min.
Is to the ſine of the half ſum 93 degr. 15 min.
So is the ſine of the Difference 15 degr. 15 min.
To another ſine, *viz.* to the ſine of 25 deg. 33 min. Unto which ſine if you adde the ſine of 90 degr. (or Radius) half that ſum ſhall be the ſine of an Arch, whoſe Complement being doubled, is the hour from the Meridian 97 degr. 54 min.

I.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 38 deg. 30 min. the Complement of the Latitude, the ſame extent will reach from 78 deg. the Complement of the Altitude, to 37 deg. 30 min.

II.

Extend the Compaſſes from 37 deg. 30 min. to the ſine of the half ſum 93 deg. 15 min. (or its Complement 86 deg. 45 min.) the ſame extent will reach from the ſine of 15 degr. 15 min. the Difference, to the ſine of 25 deg. 33 min.

Divide the diſtance between 25 deg. 33 min. and 90 deg. into two equal parts upon the Line, and the Compaſs point will reſt upon 41 deg. 3 min. the Complement whereof is 48 deg. 57 min. whoſe double 97 deg. 54 min. is the hour from Midnight (or from the North part of the Meridian) which converted into to time (by allowing 15 deg. for on hour, and 4 deg. for one min. of time) will be 6 hours 31 min. So that if the time were in the Morning, the hour would be 31 min. after 6, but if in the Evening 29 min. after 5.

Probl. XIII.

The Declination 20 *deg.* Altitude 12 *deg. and* Azimuth 146 *deg. of the Sun, being given, to find the hour of the day.*

The Proportion is,

As the co-ſine of the Declination 70 deg.

Is to the ſine of the Azimuth 146 deg. or 34 deg.

So is the co-ſine of the Altitude 78 deg.

To the ſine of the hour ſtom Noon 35 deg. 36 min.

Extend the Compaſſes from the ſine of 70 deg. (the Complement of the Suns Declination) to the ſine of 146 deg. (the Azimuth from the North) or to its Complement to 180 deg. *viz.* 34 deg. the ſame extent will reach from 78 deg. the Complement of the Suns Altitude) to the ſine of 35 deg. 36 min. the time from Noon; which converted into time is 2 hours and 22 min.

Probl. XIV.

The Suns Declination 20 deg. *his* Altitude 12 *deg. and the hour from Noon* 35 *deg.* 36 *min. being given, to find the Suns* Azimuth *from the North part of the Meridian.*

The Proportion is,

As the co-ſine of the Altitude 78 degr.

Is to the Sine of the hour from Noon 35 degr. 36 min.

So is the co-ſine of the Suns Declination 70 deg.

To the ſine of the Azimuth from the North part of the Meridian 146 deg. or 34 degr. from the South.

Extend the Compaſſes from the ſine of 78 deg. (the Complement of the Altitude) to the ſine of 35 deg. 36 min. (the time from Noon) the ſame extent will reach from 70 deg. (the Complement of the Suns Declination) to the ſine of 146 deg. (the Azimuth from the North) or 34 deg. the Azimuth from the South part of the Meridian.

Probl. XV.

The hour from Noon 35 deg. 36 min. the Latitude *of the Place 51 deg. 30 min. and the* Altitude *of the Sun 12 deg. being given, to find the angle of the* Suns Poſition.

The Proportion is,

As the co-ſine of the Suns Altitude 78 deg.

Is to the ſine of the hour from Noon 35 degr. 36 min.

So is the co-ſine of the Latitude 38 deg. 30 min.

To the ſine of the angle of the Suns Poſition at the time of the Queſtion (21 degr. 45 min.)

Extend the Compaſſes from 78 deg. the Complement of the Suns Altitude (to the ſine of 35 deg. 36 min. (the time from Noon) the ſame extent will reach from 38 deg. 30 min. (the Compliment of the Latitude) to the ſine of 21 deg. 45 min. (the angle of the Suns Poſition.)

Probl. XVI.

The Suns Altitude 12 *deg. his* Declination 20 *deg. and* Azimuth *from the* North 146 *deg. being given, to find the* Latitude.

The Proportion is,

As the Sine of the Suns Azimuth 146 deg. (or 34 deg.)

Is to the ſine of the Suns diſtance from the North-pole 110 deg. (or 70 deg.)

*S*o is the ſine of the angle of the Suns poſition 21 deg. 45 min.

To the Complement of the Latitude 38 deg. 30 min.

Extend the Compaſſes from 146 deg. (the Azimuth from the North, or from 34 deg. the Azimuth from the South) to the ſine of 110 deg. (the *S*uns diſtance from the North-pole, or to 70 deg. its diſtance from the South-pole) the ſame extent will reach from 21 deg. 45 min. (the angle of the Suns poſition) to 38 deg. 30 min. (the Complement of the Latitude) which taken from 90 deg. leaves 51 deg. 30 min. for the Latitude it ſelf.

Probl. XVII.

The Latitude of the Place 51 *deg.* 30 *min. the* Suns Declination 20 *deg. North, and the* Hour *from Noon* 4 (viz. 60 *deg.*) *given, to find the* Suns Altitude.

The Proportions are,

(1.) As Radius 90 deg.

To ſine 30 deg. the Complement of the hour from Noon,

So is the Co-tangent of the Latitude 38 deg. 30 min.

To a fourth Tangent, 21 deg. 41 min.

Which taken from 70 deg. the Complement of the Suns Declination, there reſts 48 deg. 18 min. for a fifth ſine. Then,

(2.) As the Coſine of the fourth ſine 68 deg. 18 min.

To the Coſine of the fifth ſine 4 deg. 42 min.

So is the ſine of the Latitude 51 deg. 30 min.

To the ſine of 34 deg. 5 min. the Suns Altitude at 8 in the morning, or 4 in the afternoon.

Extend the Compaſſes from the ſine of 90 deg. to ſine 30 deg. the ſame will reach from the tangent of 38 deg. 30 min. to the tangent of 21 deg. 41 min. which taken from 70, reſts 48 deg. 18 min. — Again, Extend the Compaſſes from 68 degr. 18 min. to the ſine of 41 deg. 42 min. the ſame extent will reach from the ſine of 51 deg. 30 min. to the ſine of 34 deg. 5 min. the Suns Altitude at 8 or 4 a clock.

The USE of the PROPORTIONAL LINES IN *DIALLING.*

CHAP. VI.

SECT. I.

Of the diſtinction of Plains ; *upon which* Dials *are uſually made.*

ALL Plains upon which *Dials* are uſually made, are one of theſe three ſorts, *viz.* either

Parallel
Perpendicular } to the Horizon.
Oblique

1. If the Plain be *Parallel* to the Horizon, it is called an *Horizontal Plain*; and of theſe Plains there is no variety.

2. If the Plain be *Perpendicular* to the Horizon, it is called a *Vertical Plain*, and if the face of the Plain do directly behold the true *Eaſt*, *Weſt*, *North*, or *South* points of the Horizon, they are called the *Direct Verticals*; but if they lie between any of the four fore-mentioned *Cardinal Points*, they are then called *Vertical Plains declining* from the true *North* or *South* Points, towards either the *Eaſt* or *Weſt*.

3. If the Plain lie *Obliquely* to the Horizon, and be neither *Horizontal* nor *Vertical*, then they are called *Reclining* or *Inclining* Plains; *Reclining* from the *Zenith*, or *Inclining* towards the *Horizon*: and of theſe there are ſeveral varieties in reſpect of their *Reclination*, *Inclination*, or *Declination*.

4. The *Declination* of a Plain is an *Arch of the Horizon comprehended between the true* $\begin{matrix}\text{North}\\ \text{South}\end{matrix}$ Point, and a line iſſuing perpendicular from the Plain.

5. The $\left\{\begin{matrix}\textit{Reclination}\\ \textit{Inclination}\end{matrix}\right\}$ is an Arch of a Vertical Circle comprehended between the $\left\{\begin{matrix}\textit{Zenith}\\ \textit{Horizon}\end{matrix}\right\}$ and the Plain.

How to find out the *Quantity* of theſe, and alſo of ſeveral other *affections* belonging to ſeveral Plains, I have handled at large elſewhere, and ſhall ſay nothing of them in this place; my buſineſs here being (not to teach the whole Art of Dialling) but to ſhew the Uſe of the *Proportional Lines*.

SECT. 2.

SECT. 2.

To find all the Requiſites *belonging to any* Sun-Dial *by the* Proportional Lines.

I. *For an* Horizontal *Dial.*

IN the making of theſe Dials, there is nothing required but the height of the Pole or Stile above the Plain, and that is alwayes equal to the *Latitude* of the Place.

II. *In a Direct* Vertical North *or* South Plain.

In theſe Plains alſo there is nothing required but the height of the Pole or Stile above the Plain, and that is alwayes equal to the *Complement* of the *Latitude* of the Place.

III. *In* Vertical declining *Plains.*

In theſe Plains (before the Hour Lines can be drawn upon them) three things (beſides the *Latitude of the Plain* and the *Plains Declination*) muſt be found: *viz.*

1. *The height of the Pole above the Plain.*

2. *The diſtance of the* Subſtile *from the Meridian.*

3. *The Plains difference of Longitude.*

Example, In a Vertical Plain declining from the North or South 30 degr.

1. *For the height of the* Stile.

The Proportion is,

As Radius

To the Coſine of the Latitude 38 deg. 30 min.

So is the Coſine of the Declination of the Plain 60 deg.

To the ſine of the Stiles height 32 deg. 37 min.

Extend the Compaſſes from the

ſine of 90 deg. to the ſine of 38 deg. 30 min. the ſame extent will reach from the ſine of 60 , to the ſine of 32 deg. 37 min. for the height of the Stile.

2. *For the Subſtiles diſtance from the Meridian.*

The Proportion is,

As Radius 90 deg.
Is to the ſine of the Plains Declination 30 deg.
So is the Co-tangent of the Latitude 38 deg. 30 min.
To the tangent of the diſtance of the Subſtile from the Meridian, 21 deg. 41 min.

Extend the Compaſſes from 90 deg. to the ſine of 30 deg. the ſame will reach from the tangent of 38 degr. 30 min. to the tangent of 21 deg. 41 min. the diſtance required.

3. *For the Plains difference of Longitude.*

The proportion is,

As the Coſine of the Latitude 38 deg. 30 min.
Is to the Radius 90 degr.
So is the ſine of the Subſtiles diſtance from the Meridian 21 deg. 41 min.
To the ſine of 36 deg. 25 min.

Extend the Compaſſes from the ſine of 38 deg. 30 min. to the ſine of 90 deg. the ſame will reach from the ſine of 21 deg. 40 min. to the ſine of 36 deg. 25 min. for the Plains difference of Longitude.

IV. *In* Eaſt *or* Weſt Reclining *or* Inclining Plains.

In theſe Plains (as in Upright or Vertical Decliners) three things (beſides the Latitude and Reclination) muſt be known, before the Hour diſtances can be calculated; and thoſe are the ſame as in the other.

Example, In a direct Eaſt or Weſt Plain, Reclining from the Zenith 35 deg. in the Latitude of 51 degr. 30 min.

1. *For the height of the Stile.*

The proportion is,

As Radius 90 deg.

To the ſine of the Latitude 51 deg. 32 min.

So is the ſine of the Reclination 35 deg.

To the ſine of 26 deg. 41 min.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 51 deg. 32 min. the ſame extent will reach from the ſine of 35 deg. to the ſine of 26 deg. 41 min. the height of the Pole or Stile above the Plain.

2. *For the Subſtiles diſtance from the Meridian.*

The Proportion is,

As Radius 90 deg.
To the Coſine of the Reclination 55 deg.
So is the tangent of the Latitude 51 deg. 32 min.
To the tangent of 45 deg. 52 min.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 55 deg. the ſame extent will reach from the tangent of 51 deg. 32 min. to the tangent of 45 deg. 52 min. the Subſtiles diſtance from the Meridian.

3. *For the difference of Longitude.*

The Proportion is,

As the ſine of the Latitude 51 deg. 32 min.
To Radius 90 deg.

So is the ſine of the Subſtiles diſtance from the Meridian 45 deg. 52 min.

To the ſine of 66 deg. 27 min.

Extend the Compaſſes from the ſine of 51 deg. 32 min. to the ſine of 90 deg. the ſame extent will reach from the ſine of 45 deg. 52 min. to rhe ſine of 66 deg. 27 min. the Plains difference of Longitude.

V. *In South and North Declining* $\left.\begin{array}{l}\textit{Reclining}\\\textit{Inclining}\end{array}\right\}$ *Plains.*

In theſe Plains (beſides the Latitude of the Place the Declination and $\left\{\begin{array}{l}\text{Reclination}\\\text{Inclination}\end{array}\right\}$ of the Plain) four things muſt be found before the Hour-lines can be drawn, *viz.*

1. *The diſtance of the Meridian and Horizon.*

2. *The height of the Pole or Stile.* ☚

3. The diſtance of the Subſtile and Meridian.

4. The Plains difference of Longitude.

I. *In* South Decliners Reclining.

Let an Example be, In a Plain declining from the South Eaſterly 30 degrees, and reclining from the Zenith 55 deg. in Latitude 51 deg. 32 min.

1. For the diſtanse of the Meridian from the Horizon.

The Proportion is,

As Radius ſine 90 deg.
To the ſine of the Reclination 55 deg.
So is the tangent of the Declination 30 deg.
To the tangent of 25 deg. 19 min.

Extend the Compaſſes from 90

deg. to the ſine of 55 deg. the ſame will reach from the tangent of 30 deg. to the tangent of 25 deg. 19 min. the Complement whereof 64 deg. 41 min. is the diſtance of the Meridian and Horizon.

2. *For the Stiles height above the Subſtile.*

The Proportion is,

(1.) As Radius 90 deg.

To the ſine of the diſtance of the Meridian and Horizon 64 deg. 41 min.

So is the Co-ſine of the Reclination 35 deg.

To the Sine of 31 deg. 14 min.

Which being ſubſtracted from 51 deg. 32 min. there remains 20 deg. 18 min. Then ſay,

(2.) As the ſine of the diſt. of Merid. and Horizon 64 deg. 41 min.

To the ſine of 20 deg. 18 min.

So is the Co-ſine of the Declination 60 deg.

To the ſine of 19 deg. 25 min.

Extend the Compaſſes from ſine 90 deg. to the ſine 64 deg. 41 min. the ſame will reach from 35 deg. to the ſine of 31 deg. 14 min. which taken from the Latitude 51 deg. 32 min. leaves 20 deg. 18 min. Then, Extend the Compaſſes from 64 deg. 41 min. to ſine 20 deg. 18 min. the ſame will reach from ſine 60 deg. to the ſine of 19 deg. 25 min. for the height of the Pole or Stile above the Plain.

3. *For the diſtance of the Subſtile and Meridian.*

The Proportion is,

As the Co-tangent of the Declination 60 deg.

To the tangent of the height of the Pole above the Plain 19 deg, 25 min.

So is the ſine laſt found 31 deg. 14 min.

To the ſine of 6 deg. 2 min.

Extend the Compaſſes from the tangent of 60 deg. to the tangent of 19 deg. 25 min. the ſame extent wil reach from the ſine of 31 deg. 14 min. to the ſine of 6 deg. 2 min. the Subſtiles diſtance from the Meridian.

4. *For the Plains difference of Longitude.*

The Proportion is,

As the ſine of 20 deg. 18 min. (the difference of the firſt found Arch and the Latitude)
To ſine 90 deg.
So is the ſine of the Subſtiles diſtance from the Meridian 6 deg. 2 min.
To the ſine of 17 deg. 38 min.

Extend the Compaſſes from the ſine of 20 deg. 18 min. to ſine 90 deg. the ſame extent will reach from the ſine of 6 deg. 2 min. to the ſine of 17 deg 38 min. the Plains difference of Longitude.

§ II. *In* North Decliners Reclining.

In theſe Plains (as in the South Decliners Reclining) the ſame four things muſt be found before the Hour diſtances can be obtained: Wherefore,

Let our Example be of a North Plain declining Weſterly 60 deg. and Reclining 54 deg.

1. *For the diſtance of the Meridian from the Horizon.*

The Proportion is,

As Radius 90 deg.

To the ſine of Reclination 54 deg.

So is the tangent of Declination 60 deg.

To the tangent of 54 deg. 29 min. which taken from 90 degr. leaves 35 deg. 31 min.

Extend the Compaſſes from ſine 90 deg. to ſine 54 deg. the ſame will reach from the tangent of 60 degr. to the tangent of 54 degr. 29 min. whoſe Complement to 90 deg. is 35 deg. 31 min. the diſtance of the Meridian and Horizon.

2. *For the Stiles height above the Subſtile.*

The Proportion is,

(1.) As the ſine of the Declination 60 deg.

To ſine 90 deg.

So is the Co-ſine of the Meridians diſtance from the Horizon, 54 deg. 29 min.

To the ſine of 70 deg. 2 min. for a firſt arch. —— To which adde 38 deg. 28 min. the Complement of the Latitude, the ſum will be 108 deg. 30 min. whoſe Complement to 180 deg. is 71 deg. 30 min. for a ſecond Arch.

(2) As the ſine of the firſt Arch 70 deg. 2 min.
To the Sine of the Reclination 54 deg.
So is the Sine of the laſt found Arch 71 deg. 30 min.
To the ſine of 54 deg. 43 min.

Extend the Compaſſes from the ſine of 60 deg. to the ſine of 90 deg. the ſame will reach from the ſine of 54 deg. 29 min. to the ſine of 70 deg. 2 min. —— To which 38 deg. 32 min. being added, makes 108 deg. 30 min. whoſe Complement to 180 deg. is 71 deg. 30 min. Then — Extend the Compaſſes from ſine 70 deg. the firſt Arch, to ſine 54 deg. (the Reclination) the ſame extent will reach from the ſine of the ſecond found arch 71 deg. 30 min. to ſine 54 deg. 43 min. for the height of the Stile.

3. *For the Subſtiles diſtance from the Meridian.*

The Proportion is,

As the tangent of the Reclination 54 deg.
Is to the ſine of 54 deg. 29 min. (the firſt found arch)
So is the tangent of the Stiles height 54 deg. 43 min.
To the ſine of 56 deg. 42 min.

Extend the Compaſſes from the tangent of 54 deg. to the ſine of 54 deg. 29 min. the ſame will reach from the tangent of 54 deg. 43 min. to the ſine of 56 deg. 42 min. Which 56 deg. 42 min. (or 123 deg. 18 min. the Complement thereof to 180 deg.) is the Subſtiles diſtance from the Meridian, according as you will account it from North or South.

4. *For the Plains difference of Longitude.*

The Proportion is,

As the ſine of the Stiles height 54 deg. 43 min.
To the ſine of 90 deg.
So is the tangent of the diſtance of the Subſtile and Meridian 56 deg. 42 min.
To the tangent of 61 deg. 48 min.

Extend the Compaſſes from the ſine of 54 deg. 43 min. to the ſine of 90 deg. the ſame will reach from the tangent of 56 degr. 42 min. to the tangent of 61 deg. 48 min. the Complement of the Plains difference of Longitude from the North, or its Complement to 180 deg. counted from the South.

SECT. 3.

Of Direct North and South Reclining Plains.

IN Direct North and South Reclining Plains there is nothing required before the Hour diſtances can be calculated, but the height of the Pole or Stile above theſe Plains; and that may be eaſily thus found.

1. *South Recliners.*

If the Reclination of the Plain be $\left\{\begin{array}{c}\text{leſs}\\ \text{more}\end{array}\right\}$ then the Complement of the Latitude $\left\{\begin{array}{c}\text{ſubſtract}\\ \text{adde}\end{array}\right\}$ the Reclination $\left\{\begin{array}{c}\text{from}\\ \text{to}\end{array}\right\}$ the Complement of the Latitude, and the $\left\{\begin{array}{c}\text{Remainer}\\ \text{Sum}\end{array}\right\}$ will be the height of the Pole or Stile above the Plain.

2. *Of North Recliners.*

The Complement of the Latitude and Reclination added together, gives the height of the Pole or Stile above the Plain: — But if this ſum do exceed 90 deg. ſubſtract it from 180 deg. and the remainer ſhall be the height of the Pole or Stile above the Reclining Plain.

SECT. IV.

To calculate the Hour diſtances upon all ſorts of Plains.

UPon all *Horizontal*, *Direct North* and *South* Plains, whether *Erect* or *Reclining*, the *Stile* ſtands upon 12 *of the Clock*; for theſe Plains have no difference of Longitude; and therefore the *Subſtile* and *Hour-line* of 12 are the ſame; and the Equinoctial diſtance of each hour from 12 is 15 deg.

In all other *Plains*, as *Erect Decliners*, and *North and South Recliners declining*, which have *difference of Longitude*, the Plains difference of Longitude muſt be reduced into Time, allowing 15 deg. for an hour, *i. e.* one degree for 4 min. of time; and ſo having found the Equinoctial diſtance of the two hours on either ſide of the Subſtile, the Equinoctial diſtances for all the other hours are eaſily found, by the continual addition of 15 degrees for one hour: And then the proportion will be,

As the ſine of 90 deg.

To the ſine of the height of the Pole or Stile above any Plain whatſoever it be;

So is the tangent of the Equinoctial diſtance of any hour from the Meridian of any Direct North or South Plain, whether Erect or Reclining

clining: — But from the Subſtile of all other Plains.

To the tangent of the true hours diſtance upon the Plain, from the Meridian or Subſtile.

This Analogy is general for all ſorts of Plains, and ſo let this ſuffice for DIALLING *in this place.*

The

The USE of the PROPORTIONAL LINES IN *GEOGRAPHY.*

CHAP. VII.

Probl. I.

Two Places which differ only in Latitude, to find their Diſtance.

I. IF both the Places lie under one and the ſame Meridian, and on one and the ſame ſide of the Æquinoctial, ſubſtract the leſſer Latitude from the greater, and the Difference converted into Miles (by allowing 60 Miles to one degree) ſhall give you the diſtance.

Example, *London* and *Ribadio* lie both under one Meridian, but differ in Latitude, for *London* hath 51 deg. 30 min. and *Ribadio* Latitude 43 deg. both North; the difference of Latitude is 8 degr. 30 min. which being turned into miles makes 510 miles.

2. If the two places lie under one and the ſame Meridian, but one on the North, and the other on the South ſide of the Æquinoctial, add both the Latitudes together, the ſum is the diſtance.

Example, *London* and the Iſland *Triſtan Dacunhu* lie both under one Meridian, but *London* hath 51 deg. 30 min. North Latitude, and the Iſland hath 34 deg. South Latitude; their ſum is 85 degr. 30 min. which converted into miles (by dividing the degrees by 60, and allowing for every minute one mile) makes 5130 miles; and ſuch is the diſtance of *London* and the Iſland *Triſtan Dacunhu*.

Probl. 2.

Two places which differ only in Longitude, to find their Diſtance.

1. THe two places may lie both under the Æquinoctial, and have no Latitude; in this Caſe the difference of their Longitudes (if it be leſs than 180 degr.) reduced into miles is their Diſtance; but if their difference exceed 180 degr. take it out of 360 degr. the remaining degrees turned into miles will be the Diſtance of the two Places.

Example, The Iſland *Sumatra* and the Iſland of St. *Thoma* lie both under the Æquinoctial, the Iſland of Sr. *Thoma* having 33 deg. 10 min. of Longitude, and the Iſland *Sumatra* 137 degr. 10 min. The leſſer Longitude taken from the greater leaves 104 deg. 0 min. which converted into miles is 6240; and that is the diſtance of the two Iſlands.

2. But if the two places differ only in Longitude, and lie not under the Æquinoctial, but under ſome other intermediate Parallel of Latitude: As *Conſtantinople* and *Compoſtella*, both in the Latitude of 47 degr. but differing in Longitude 43 deg. 15 min. then

The Proportion is,

As the Radius 90 degr.
Is to the Co-ſine of the common Latitude 47 degr.
So is the ſine of half the difference of Longitude 21 deg. 37 min.
To the ſine of half their Diſtance 15 degr. 38 min.

Extend the Compaſſes from the ſine of 90 deg. to 47 deg. (the Complement of the Common Latitude) the ſame extent will reach from the ſine of 21 deg. 37 min. (half the difference of Longitude) to the ſine of 15 deg. 38 min. half the diſtance of the two places.

The double whereof is 31 deg: 16 min. or 1876 miles.

Probl. 3.

Two places differing both in Longitude and Latitude being propoſed, to find their diſtance.

1. ONe of the places may lie under the Æquinoctial, and have no Latitude, and the other under ſome Parallel of Latitude between the Æquinoctial and one of the Poles. For finding the diſtance of places that are ſo ſituate, as St. *Thome Iſland* under the Æquinoctial in 33 deg. 10 min. Longitude, and *Lon-*

don in 51 deg. 30 min. North Latitude, and Longitude 20 deg. the difference of Longitude being 13 deg. 10 min.

The Proportion is,

As the Radius 90 deg.
Is to the Co-ſine of the difference of Longitude 76 degrees 50 min.
So is the Co-ſine of the Latitude given 38 deg. 30 min.
To the Co-ſine of the diſtance required, 52 degr. 41 min.

Extend the Compaſſes from the ſine of 90 degr. to the ſine of 76 degr. 50 min. (the Complement of the difference of Longitude) the ſame extent will reach from 38 deg. 30 min. (the Complement of the given Latitude) to 37 degr. 19 min. (the complement of the diſtance of the places) that is 52 deg. 41 min. which in miles is 3161.

2. If both the Places propoſed ſhall be without the Æquinoctial, but on one ſide, either both towards the North, or both towards the South, as *London*, in Longitude 20 deg. and Latitude North 51 deg. 30 min. and *Jeruſalem* in Longitude 66 deg. and Latitude North 31 deg. 40 min. the difference of Longitude being 46 deg. uſe this proportion:

(1.) As Radius 90 deg.

Is to the ſine of 44 deg. the complement of the difference of Longitude,

So is the tangent of the complement of the greater Latitude 38 deg. 30 min.

To the tangent of 28 deg. 55 min. a fourth Term.

This 28 deg. 55 min. being taken from the complement of the leſſer Latitude 58 deg. 20 min. there remains 29 deg. 24 min. Then, (2.) As the ſine of 28 deg. 55 min. the fourth Term,

Is to the Co-ſine of the greater Latitude 38 deg. 30 min.

So is the ſine of the Remainer 29 deg. 24 min.

To the ſine of 39 deg. 11 min. the diſtance.

(1.) Extend the Compaſſes from the ſine of 90 deg. to the ſine of 44 deg. (the complement of the difference of Longitude) the ſame extent will reach from the tangent of 38 deg. 30 min. (the complement of the Latitude of *London*) to the tangent of 28 deg. 56 min. for a fourth number; which taken from the complement of the leſſer Latitude *Jeruſalem* 58 deg. 20 min. leaves 29 deg. 24 min.

Again,

(2.) Extend the Compaſſes from the ſine of 28 deg. 55 min. the fourth number, to the ſine of 38 deg. 30 (the complement of the greater Latitude *London*) the ſame extent will reach from 29 deg. 24 min. (the former remainer) to the ſine of 39 deg. 11 min. for the diſtance of the two places; which in miles is 2331.

3. The two places propounded may be ſo ſituate, that one may be in North Latitude, the other in South, and be under different Longitudes: As ſuppoſe the places to have

	deg.	*min.*
Latitude N	50	0
Latitude S	32	25

And differ in Longitude 70 deg. ſay,

(1.) As Radius

To the ſine of 20 deg. the complement of the difference in Longitude;

So is the tangent of the greater Latitude 50 degr.

To the tangent of 16 deg. 1 min.

Take this fourth Term 16 deg. 1 min. from 57 deg. 35 min. the complement of the leſſer Latitude, and the remainer will be 41 deg. 34 min.

And ſay again,

(2.) As the ſine of 73 deg. 59 min. (the complement of the fourth Term before found)

To 48 deg. 26 min. (the complement of the Remainer,)

So is the ſine of 50 deg. (the greater Latitude,)

To the ſine of 36 deg. 36 min. (whoſe complement 53 deg. 24 min.) is the diſtance, which in miles is 3205.

The USE of the PROPORTIONAL LINES IN NAVIGATION.

CHAP. VIII.

THe principal Problems in uſe with Mariners in their Navigations (beſides thoſe of Aſtronomy and Geography in the foregoing Chapters) are ſuch as concern *Longitude*, *Latitude*, *Rumb*, and *Diſtance*, a few of which I ſhall ſhew how to perform by the *Proportional Lines*.

Example

Examples in Figure I.

In which Figure,

CA repreſent the Meridian, *C* North and *A* South.

BA, A Parallel of Latitude, *B* Weſt and *A* Eaſt.

CB, A Rumb 53 deg. 7 min. diſtant from the Meridian Weſtward, which Rumb is N. W. 8 deg. 7 min. Weſterly from *C*. and N. E. by N. 19 min. Eaſterly.

And ſo

CB Is the Courſe or Rumb.

CA The difference of Latitude, and

BA The departure from your firſt Meridian.

Probl. I.

Probl. 1.

The courſe and diſtance given, to find the difference of Latitude and departure from your firſt Meridian.

SAiling from C 225 min the Courſe or Rumb is N W 8 deg. 7 min. Weſterly (that is 53 deg. 7 min. from the Meridian) I demand how much I have altered my Latitude, and how far I have departed from my firſt Meridian.

The Proportion is,

As Radius 90 deg.

Is to the diſtance ſailed 225 min.

So is the ſine of the Rumb 53 deg. 7 min.

To 180 the departure from your firſt Meridian.

And

So is the complement of the Rumb 36 deg. 53 min.

To 135 min. the difference of Latitude 1.

Extend the Compaſſes from the ſine of 90 deg. to 225, the ſame extent will reach from 53 deg. 7 min. the Rumb, to 180 min. for your departure: —— And alſo the ſame extent will reach from 36 deg. 53 min. the complement of the Rumb, to 135 min. for the difference of Latitude.

Probl. 2.

The courſe and difference of Latitude given, to find the diſtance ſailed, and the departure from your firſt Meridian.

LEt the Courſe be N. W. 8 deg. 7 min. Weſterly (or 53 deg. 7 min. from the Meridian) as before; the difference of Latitude 135 min. and let the diſtance ſailed CB, and the departure B A be required.

The Proportion is,

As the Co-ſine of the Courſe 36 deg. 53 min.

Is to 135 min. the difference of Latitude;

So is Radius 90 deg.

To 225 min. the diſtance ſailed,

And

So is the ſine of the Rumb 53 deg. 7 min.

To 180 min. the departure.

Extend the Compaſſes from 36 deg. 53 min. to 135, the ſame extent will reach from 90 deg. to 225 for the diſtance ſailed: — And from 53 deg. 7 min. to 180 min. the departure from your Meridian,

Probl. 3.

Probl. 3.

The courſe and departure being given, to find the diſtance ſailed and the difference of Latitude.

LEt the courſe be N. W. 8 deg. 7 min. Weſterly (or 53 deg. 7 min. from the Meridian) and the departure from the Meridian 180 min. and let the diſtance ſailed and the difference of Latitude be required.

The proportion is,

As the ſine of the courſe 53 deg. 7 min.

Is to the departure 180 min.

So is Radius 90 degr.

To 225 the diſtance ſailed.

And

So is the complement of the courſe 36 deg. 53 min.

To 135 min. the difference of Latitude.

Extend the Compaſſes from the ſine of 53 deg. 7 min. to 180 min. the ſame extent will reach from the ſine of 90 deg. to 225 min. the diſtance ſailed; and tho ſame extent alſo will reach from 36 deg. 53 min. the complement of the courſ, to 135 min. the difference of Latitude.

Probl. 4.

The difference of Latitude and diſtance ſailed, given, to find the courſe and departure from the Meridian.

A Ship ſails between the North and the Weſt 225 min. ſo long till ſhe hath altered her Latitude 135 min. I demand what courſe the Ship hath made, and alſo how far ſhe hath departed from her firſt Meridian.

The Proportion is,

As the ſine of 90 degr.
Is to 225 m. the diſtance ſailed,
So

So is 135 min. the difference of Latitude,

To 36 deg. 53 min. the complement of the courſe that the Ship ſailed.

And

So is the ſine of 53 deg. 7 min.

To 180 min. the departure.

Extend the Compaſſes from 225 min. to the ſine of 90 deg. the ſame extent will reach from 135 min. to 36 deg. 53 min. whoſe complement 53 deg. 7 min. is the courſe. — And the ſame extent alſo will reach from 53 deg. 7 min. to 180 min. the Ships departure from the firſt Meridian.

Probl. 4.

The diſtance and departure given, to find the courſe and difference of Latitude.

THe diſtance ſailed is 225 min. and the departure is 180 min. I demand the courſe and difference of the Latitude: For which

The proportion is,

As 225 the diſtance ſailed,
Is to the ſine of 90 deg.
So is 180 the departure,
To the ſine of 53 deg. 7 min. the courſe,

And

So is the ſine of 36 deg. 53 min. the Complement of the Courſe,
To 135 min. the difference of Latitude.

Extend the Compaſſes from 225 min. the diſtance, to 90 deg. the ſame extent will reach from 180 min. the difference, to 53 deg. 7 min. the courſe, which is N. W. 8 deg. 7 min. Weſterly —— And the ſame extent will reach from 36 deg. 53 min. the complement of the courſe, to 135 min. the difference of Latitude.

Probl. 5.

The diffeeence of Latitude and departure given, to find the courſe and diſtance.

THe difference of Latitude is 135 min. and the departure is 180 min. the Rumb and Diſtance is required:

The Proportion is,

As 135 min. the difference of Latitude,

Is to Radius (or Tangent of 45 degr.)

So is 180 min. the diſtance,

To the tangent of 53 deg. 7 min. the courſe.

And

So is the tangent of 45 deg.

To 225 min. the diſtance ſailed.

Extend the Compaſſes from 135 min. the difference of Latitude, to the tangent of 45 deg. the ſame extent will reach from 180 min. the diſtance, to the tangent of 53 deg. 7 min. or N. W. 8 deg. 7 min. Weſterly for the courſe. —— And the ſame extent alſo will reach from the tangent of 45 deg. to 225 min. diſtance ſailed.

FINIS.

THE LINE OF PROPORTION,

Petiver

Commonly called

5295 3

GUNTER'S LINE,

Made Eaſie.

A SECOND PART.

With the addition of other Lines, which may conveniently be put upon a *Two-foot Rule*, and their **USES** Exemplified, In

Arithmetick, *Geometry*, *Military Affairs*, *Trigonometry*,

Aſtronomy, *Dialling*, *Geography*, *Navigation*, &c.

By WIL. LEYBOURN *Philom.*

To which is added a

SUPPLEMENT,

Containing the Deſcription and ſome Uſes, of a convenient Two-foot JOYNT-RULE:

Upon which are inſcribed divers Lines and Scales, ſuitable to all ſort of Artificers occaſions.

By JOHN BROWN.

London, Printed by *W. L.* and *T. J.* for *George*
Sawbridge at the *Bible* on *Ludgate-Hill.* 1677.

MVSEVM
BRITAN
NICVM

To the READER.

THe Good Acceptance which the former Part of this BOOK *hath received in the World (which was entituled,* The Uſe of the Line of Proportion (*or* Numbers) *commonly called* Gunter's Line made eaſie) *hath animated me to write ſome other* Precepts, *and to adde ſome other* Proportional Lines *of Mr.* Gunter's *firſt contrivance from* *his* Logarithmical Tables *of* Artificial Sines *and* Tangents *upon a* Strait Ruler.

In the Firſt Part *I have principally applied the* Line of Numbers *to ſuch kind of* Menſurations *as are of daily uſe amongſt Workmen, as in the Menſuration of* Board, Glaſs, Timber, Stone, Brick-work, Tiling, Painting, Paving, Plaiſtering, Wainſcotting, &c. *of all which (and ſome other Menſurations) I have given there ſufficient Rules and Examples. Wherefore I ſhall (in this* Second Part) *omit to ſay any thing of ſuch matters or things as I have at large handled therein; although all the*

Work in that Book contained may be performed upon one of the Lines which is upon this Ruler, namely, by the Line of Numbers of two Radiuſſes; *but ſhall principally diſcourſe, or treat, of the Uſes of ſuch other* Proportional Lines *as are inſcribed upon this* Ruler, *as now contrived: And yet I will not forbear to ſhew how to perform many* Problems *in the Former Part by this* Ruler *alſo; but they ſhall only be ſuch, which by the Lines (as they are now diſpoſed) may be wrought with leſs* Trouble, *more* Speed, *and the ſame* Exactneſs; *and many, which there (by the Single Line) required greateſt* trouble *in their* performance,

formance, *may be here done with the greateſt* eaſe; *nay, many* (*and thoſe the moſt difficult*) *by inſpection only, not medling with any other*; *My principal aim in this* Second Part *being to ſhew ſuch other Uſes of the Common* (*or General*) *Line* (viz. *the* Line of Numbers) *together with ſuch other Proportional Lines or Scales upon this Ruler inſcribed, in the ſolution of the moſt uſeful and neceſſary Problems in* Arithmetick, Geometry, Aſtronomy, Geography, Navigation, Dialling, Trigonometry, *and ſeveral other of the* Mathematical Sciences, *as ſhall render it a moſt abſolute and neceſſary Concometant, not only*

for

ſor Artificers, *but for all ſorts or degrees of Men, of what quality ſoever, that are any wayes inclinable to, or delighted in* Mathematical Practices.

And in order thereunto I have under apt Heads and diſtinct Titles (and not miſcellaneouſly) given variety of Problems *and* Examples *in all the above-mentioned Sciences.*

I ſhall not ſay more to induce you to the peruſal of theſe Tractates, but commend you to the Practice of what is herein contained; and (beſides the delight you will take therein, the benefit and profit you may re-

receive thereby) will be ſufficient motives to induce you to their peruſal.

And now let me acquaint thee Reader, that unto this Second Part there is added a Supplement, *containing the* Deſcription, *and ſome* **U**ſes (*and thoſe not a few*) *of a convenient* Two-foot Joynt-Rule.

Thus have I given you a ſhort Account of what is contained both in this Second Part *and in the* Supplement ; *both which I commend unto thee, wiſhing thee good ſucceſs in thy peruſal and practice of them ; and in a ſhort time thou mayeſt expect ſome other Treatiſes of this kind,* *and of other Parts of the Mathematicks alſo (ſome of them being almoſt ready for the Preſs:) from him who wiſhes thy welfare and the Advancement of Knowledge in the Common-wealth wherein he lives.*

London,
May 21. 1677.

Will. Leybourn.

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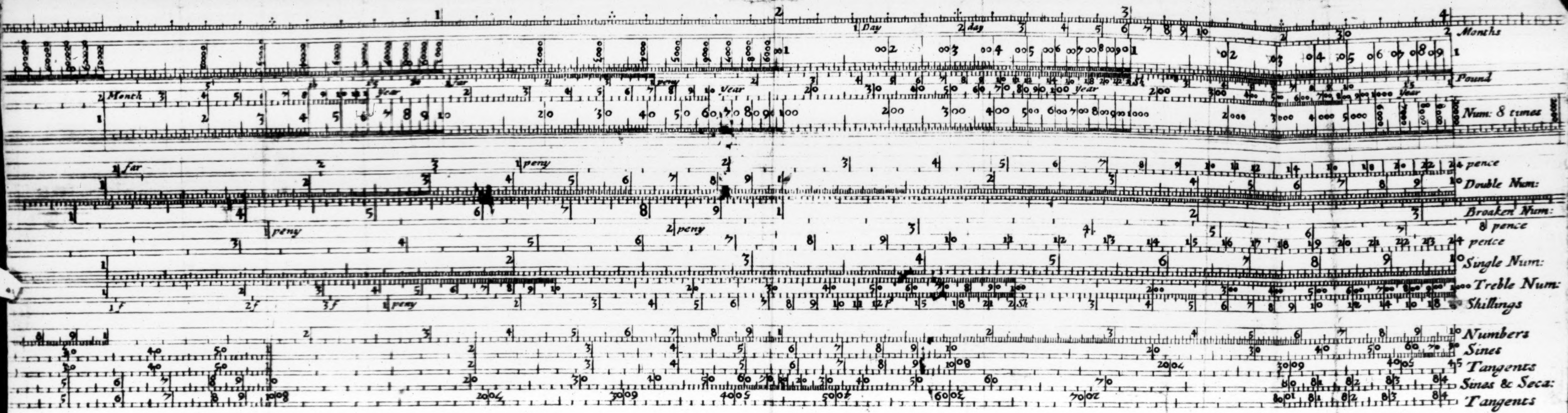
Advertiſement.

T*Heſe* RULES, *and all other* Mathematical Inſtruments, *either for Sea or Land, are made and ſold by* Walter Hayes *at the Sign of the* Croſs-Daggers *in* Moor-Fields, *near the* Popes-Head Tavern London.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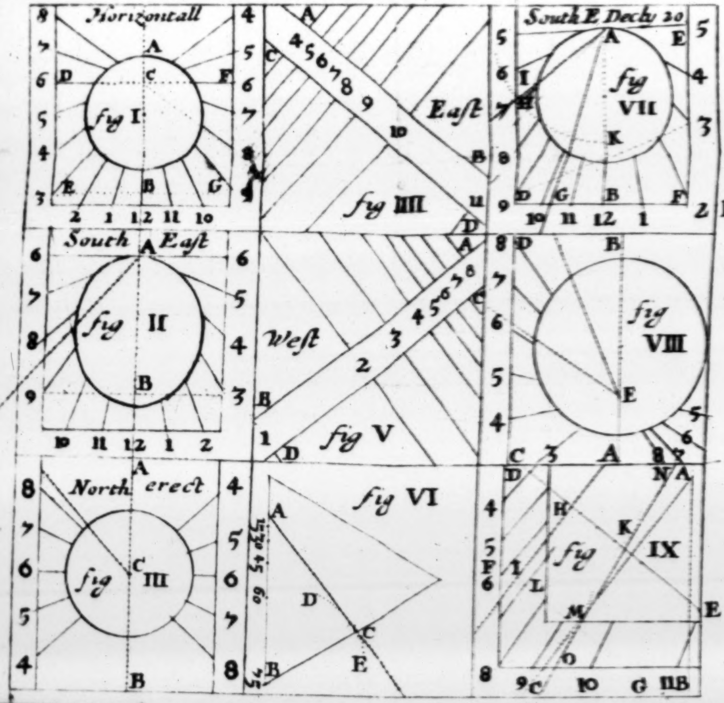
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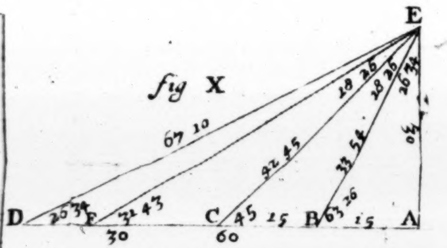

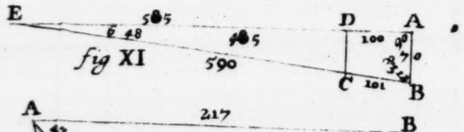

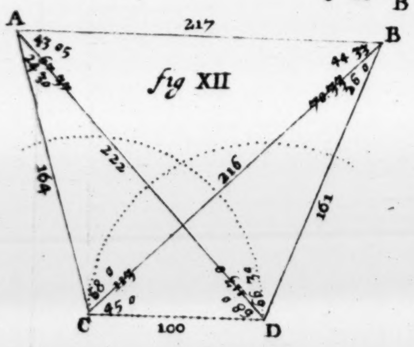

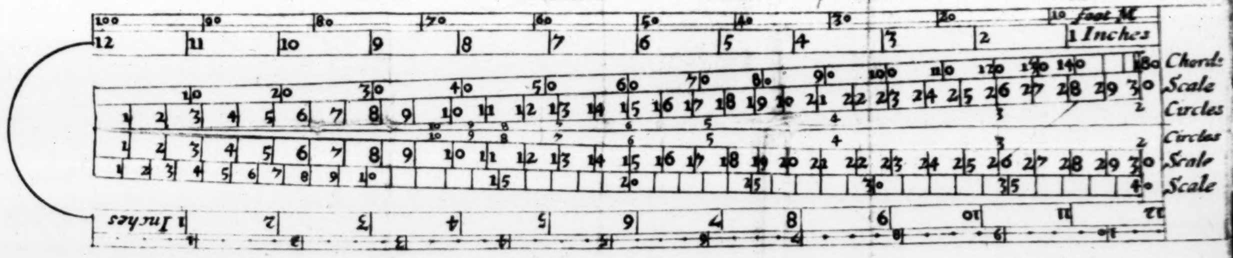
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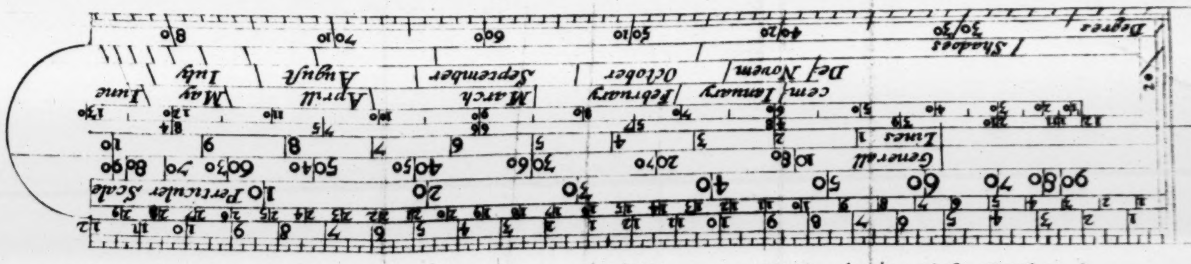
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THE LINE OF PROPORTION Made EASIE.

A SECOND PART.

CHAP. I.

he deſcription of the RULER, *and the manner how the ſeveral Lines upon it are to be diſpoſed.*

THe Ruler may be made either of Braſs, Wood or Ivory, and it may be a ſtreight Ruler of two Foot long or more, at Meaſure; or it may be in a ſtreight

Rule or Scale but of one foot lo but then ſome of the lines will be v ry ſhort, and the Diviſion on th too ſmall; Or Thirdly (and beſt all) upon a Two-foot-Joynt-Ru which opened will be the ſame a ſtreight Rule of two-foot long.

The Lines upon the Ruler are Number Eight, beſides Scales of qual parts, and of Chords, whi may be upon the edges of the R ler. But upon the flat of the Ru (as I ſaid before) Eight Scales Lines.

I. The firſt, and uppermoſt is a ſingle line of Numbers, contain the whole length (or very near) the Rule, divided firſt into ten u equal parts, and thoſe again ſu divided into ten, ſo often as qua tity will permit, according to t uſual manner of dividing of ſu Lines.

II. Next under this Line (and facing of it,) are three lines of Numbers, all of equal length, and all three of them together, are of equal length to the firſt ſingle line.

III. The third Scale is a Line of Numbers broken, having One in the middle thereof, and broken off at either end of the Rule, at 31 and 62 hundred part,

IV. Underneath this broken Line (and facing of it) is the common line of Numbers of two *Radiuſſes.*

Theſe Four fore-mentioned Lines, ſerve to *Extract* the *Square* and *Cube Roots* by Inſpection, without the uſe of Compaſſes, and for other Uſes alſo, as ſhall hereafter be made manifeſt.

V. The fifth Line is a line of *Artificial Sines*, divided into 90 unequal parts, and ſubdivided. And

Rule or Scale but of one foot lon
but then ſome of the lines will be
ry ſhort, and the Diviſion on th
too ſmall; Or Thirdly (and beſt
all) upon a Two-foot-Joynt-Ru
which opened will be the ſame a
ſtreight Rule of two-foot long.

The Lines upon the Ruler are
Number Eight, beſides Scales of
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may be upon the edges of the R
ler. But upon the flat of the Ru
(as I ſaid before) Eight Scales
Lines.

I. The firſt, and uppermoſt is
ſingle line of Numbers, contain
the whole length (or very near)
the Rule, divided firſt into ten
equal parts, and thoſe again
divided into ten, ſo often as qu
tity will permit, according to
uſual manner of dividing of ſ
Lines.

II. Next under this Line (and facing of it,) are three lines of Numbers, all of equal length, and all three of them together, are of equal length to the firſt ſingle line.

III. The third Scale is a Line of Numbers broken, having One in the middle thereof, and broken off at either end of the Rule, at 31 and 62 hundred part,

IV. Underneath this broken Line (and facing of it) is the common line of Numbers of two *Radiuſſes*.

Theſe Four fore-mentioned Lines, ſerve to *Extract* the *Square* and *Cube Roots* by Inſpection, without the uſe of Compaſſes, and for other Uſes alſo, as ſhall hereafter be made manifeſt.

V. The fifth Line is a line of *Artificial Sines*, divided into 90 unequal parts, and ſubdivided. And

VI. Is a line of *Artificial Tangen* numbered unequally to 45 Degree and back again towards 90, ſo f as the Ruler will permit.

Theſe two Lines of *Sines* and *Ta* *gents*, are both of them of o Length or *Radius*, and are to b uſed with the fourth line of Num bers of 2 *Radiuſſes*.

VII. The ſeventh Scale is a Li of *Artificial Sines*, having 90 de in the middle of the Line, and th the Diviſions are continued up b yond 90 deg. to the end of the R ler, ending at 84 deg. 10 m. or r ther at 174 deg. 10 m.

VIII. The eight Scale is a line *Artificial Tangents*, which faceth t former line of Sines, having the *R dius* (or 45 deg.) in the middle the Line, againſt 90 deg. of th *Sines*, and is continued up abov 45 deg. to the end of the Ruler

wher

where it terminates at 84 deg. 10 m. as the Sines do, and this avoids backward counting.

These two laſt lines of *Sines* and *Tangents* (being both of the ſame *Radius*) are to be uſed with the Fourth Line of *Numbers* of two *Radiuſſes*, and are of good uſe in the ſolution of *Spherical Triangles*, where Obtuſe Angles are ingredient in the Queſtion: And alſo when the *Tangent* given or required, exceeds 45 degrees.

I ſhall ſay no more concerning the Lines upon the Ruler, for every man being at liberty to inſert ſuch other as his particular occaſion ſhall require, as *Chords*, *Equal parts*, a *Meridian-line*, and ſuch like: In the Figure they are diſpoſed in this Order.

The

The USE of the PROPORTIONAL LINES IN *ARITHMETICK*

CHAP. II.

TO paſs by *Numeration*, *Mult* *plication*, *Diviſion*, the *Gold* *Rules* both *Direct* and *Reverſe*, alſo *Duplicated* and *Triplicated Pr* *portions*, they being ſufficiently trea ted of in the Seven firſt Chapters the Firſt part, I ſhall proceed the work of the Ninth Chapter which is

SEC

SECTION I.

To Extract the Square Root *by the Lines.*

THe Rule delivered for the Extraction of the Square Root in the Ninth Chapter, is, [*Divide the ſpace between* 1, *and the number whoſe Root is to be Extracted into two eqaal parts, and the middle point ſhall fall upon the Root required.*] So the root of 36 being required, if you divide the ſpace between 1 and 36 into two equal parts, the Compaſs point will reſt upon 6, which is the root of 36.—— Alſo the ſecond *Example* of that Ninth Chapter requires the root of 256, the diſtance between 1 and 256 being divided into two equal parts, the Compaſſes will fall upon 16, the root of 256.

This is the way there preſcribed, and is the only way to perform

As 100 : to 78,54 ::
So is 768, to 603,19

Extend the Compaſſes from 100, to 78,54 (a fixed Area) the ſame will reach from 768 (the Rectangled Figure made of the two Diameters) to 603,19, the Area of the Ellipſis.

Queſtion 16.

To find the Diameter of a Circle whoſe Area ſhall be equal to the Area of the former Ellipſis ?

Upon the Line of Numbers of two Radiuſſes, open the Compaſſes from 24 to 32 the two Diameters of the Ellipſis , that diſtance applied to the ſingle Line , will reach from 24 the leſſer Diameter, to 27,71 the Diameter of a Circle, whoſe Area ſhall be equal to the Area of the Ellipſis.

Queſt. 17.

The Chord Line 60,8, and Altitude 14, of the Segment of any Circle, being known, to find out the other parts of the Circle and the Area of the Circle?

1. Extend the Compaſſes from 1 to 30,4, half the Chord of the Arch, and that diſtance again repeated from 30,4, will reach to 924,16, the ſquare of half the Arch Line.

2. Extend the Compaſſes from 14 (the Altitude of the Arch) to 1, the ſame will reach from 924,16, to 66, to which if you adde 14 the Altitude of the Arch, the ſum will be 80, for the Diameter of the Circle, the half whereof 40, is the Radius of the Circle.

3. Adde half the Segments Chord 30,4, and the Segments Altitude 14, together, they make 44,4, whoſe

Square Root is 6,67 *fere*, and is the length of the Chord of half the Segments Arch.

SECT. II.

Of Solid Meaſures.

Queſt. 1.

If a piece of Square Timber be 1 inches broad, 22 inches deep, and 2 foot long: how many ſolid foot are contained therein?

Extend the Compaſſes upon th Line of two Radiuſſes, from 15 i ches the breadth, to 22 in the dept that extent ſhall reach from 15 u on the ſingle Line, to 18¼ inche for the true ſquare at the end, th your proportion will be

As 12 inches,
To the inches ſquare 18¼,

So is the length in feet 20,
To a fourth ; and that fourth to 46 foot.

Extend the Compaſſes from 12 to $18\frac{1}{4}$, the ſame will reach from 20 the length , at twice turning the Compaſſes, to 46 , the quantity of feet contained in the whole piece.

Or in Foot Meaſure.

Extend the Compaſſes from 1,25, the breadth, to 1,84 the depth, upon the Line of two Radiuſſes, that diſtance applied to the ſingle line, ſhall reach from 1, 25 to 1,52.

Again , Extend the Compaſſes from 1 to 1,52 , the ſame extent ſhall reach from 20 , at twice turning of the Compaſſes , to 46 the content of the Piece in Feet.

Queſtion 2.

If a Piece of Tapering Timber be 2,2 foot, and 0,41 foot at one end, and 1,32 foot, and 1,75 foot at the other end, and 12 foot long; how many ſolid foot is contained in this Piec of Timber?

1. Upon the Line of two Radi-uſſes, take the diſtance between 1, and 0,41, the ſame extent will reac downwards from 2,2, to 0,90, fo the content of the Baſe at the littl end.

2. Upon the ſame Line take th diſtance between 1 and 1,32, th ſame extent will reach from 1,7 to 2,31, the content of the greate end.

3. Extend the Compaſſes fro 1, downwards to 90, the Area o the leſſer Baſe, the ſame extent w reach from 2,31, the Area of th greater Baſe, to 2,08, the produ

of the two ends multiplied together, the Square Root whereof is 1,44 : Add this Root and the two Baſes together, their ſum is 4,65. Then again, Extend the Compaſſes from 1 to 4 (which is one third of the length of the piece) the ſame extent ſhall reach from 4,65, to 18,60 the true content of the whole piece.

0,90
2,31
1,44
———
4,65

Queſtion 3.

If a Cube, whoſe ſide is 12 inches, doth contain 1728 Cubical inches, how many Cubical inches ſhall a Cube contain, whoſe ſide is 8 inches?

Out of one of the Lines of 3 Radiuſſes take the diſtance from 12 to 8, the difference of ſides, that ſame diſtance applied to the ſingle Line, ſhall reach from 1728 downwards to 512, the ſolid inches in a Cube, whoſe ſide is 8 inches.

Queſtion 4.

If a Bullet, or Sphear, being 6 *inch Diameter, do weigh* 30 l. *what ſhall a Sphear of the ſame metal weigh, whoſe Diameter is* 7 *inches?*

Take the diſtance between 6 and 7 out of the Line of 3 Radiuſſes, the ſame extent applied to the ſingle line, will reach from 30, to 47,7, and ſo much will a Bullet of the ſame metal weigh, whoſe diameter is 7 inches.

Queſtion 5.

If a Ship of 300 *Tun burthen, b* 75 *foot by the Keel, what burthen ſhal that Ship be, whoſe Keel is* 100 *foot?*

The diſtance between 75 and 100 being taken out of the Line of thre Radiuſſes, applied to the ſingle Lin will reach from 300 Tun, to 71 the burthen of that Ship, whoſe Kee is 100 foot.

Question 6.

If a Ship of 300 Tun, be 29,5 foot at the Beam, what ſhall the length of the Beam of that Ship be, whoſe burthen is 713 Tun?

Out of the ſingle Line, take the diſtance between 300 and 713, that ſame extent applied to the Line of three Radiuſſes, ſhall reach from 29,5, to 39,35, for the length of the Beam of a Ship, whoſe burthen ſhall be 713 Tun.

Question 7.

If a Ship of 300 Tun be 13 foot in Hold, what ſhall that Ship be in Hold, whoſe burthen is 713 Tun?

Out of the ſingle Line, take the diſtance between 300 and 713, that diſtance applied to the Line of three Radiuſſes, ſhall reach from 13, to 17,35, and ſo much ſhall that Ship be in Hold, whoſe burthen is 713 Tun.

Queſtion 8.

If a Braſs Piece of Ordnance, whoſe Diameter is 11,5 *inches, do weigh* 1900 *pounds, what ſhall another Piece weigh, (of the ſame ſhape) whoſe Diameter is* 8,75 *inches?*

The Extent between 11,5 and 87,5 taken upon the broken Line of three Radiuſſes, will reach upon the ſingle Line, from 1900 to 837; and ſo much ſhall that Piece weigh, whoſe Diameter at Bore is 8,75.

The USE of the PROPORTIONAL LINES IN *Military Affairs.*

CHAP. IV.

SECT. I.

Queſt. I.

How to order any number of Soldiers into a Square Battail; ſo that there ſhall be as many in Rank as in File?

LET it be required to make a Square Battail of 2704 men, ſo at there be as many in Rank as in ile.

Foraſmuch as the number of Souldiers do conſiſt of an even number of Figures, ſeek that number 2704, in the firſt Radius of the Double Line of Numbers, and right againſt it in the Broken Line, you ſhall find 52, and ſo many muſt be in Rank, and as many in File: And theſe Souldiers, if they be imbattelled at Order (which is 3 Foot in Rank and as much in File) then will they occupy 24336 ſquare foot of Ground; which by the Lines you may thus find.

Extend the Compaſſes from 1 to 3 (the diſtance in Rank and File) the ſame extent will reach from 52 to 156; find 156 upon the Broken Line, and againſt it in the Double Line you ſhall find 24336, the Ground that theſe Souldiers will occupy, being at their Order of 3 foot.

Queſt.

Queſt. 2.

Any number of men being propoſed, to place them in Battalia, in ſuch order that there may be as many more in Rank as in File, and that they may ſtand at Cloſe Order, which is 1½ foot?

LEt the number given be 2602, count the half thereof 1301, upon the Double Line, and againſt it you ſhall find in the Broken Line 36, which is the depth in File, and then there muſt be 72 in Rank, which is twice 36.

Now for the Ground that theſe will occupy, being at Cloſe Order,

As 1 : is to 1,5 : : ſo is 36 : to 54: and ſo is 72 to 1080.

Extend the Compaſſes from 1 to 1,5, the ſame extent will reach from 36 (the depth of men in File) to 54 the ſide of the Ground. ---- Again, the ſame extent will reach from 72 (the Front of the men in Rank, to 1080 the length of the Ground.— Then for the Area,

Extend the Compaſſes from 1 to 1080; the ſame extent will reach from 54 to 58320, and ſo many ſquare foot of Ground will theſe 2602 men occupy at Cloſe Order.

Queſt. 3.

Any number of men being propoſed to be put in Battalia, and a certain number named to be either in Rank or File, to find the other number?

LEt it be required to place 872 men in Battalia, ſo that there ſhall be 8 in File, how many muſt there be in Rank; or how many Files muſt there be?

The Proportion to work this is,

As 8, the depth in File,
Is to 872, the number of Souldiers,

So is 1, to 109 the number of men in Rank.

Extend the Compaſſes from 3, o 87, the ſame extent (the ſame way) will reach from 1, to 109.

Queſt. 4.

Any number of Souldiers being given, together with their diſtance in Rank and File, to order them into a Square Battail of Ground?

Let the number of Souldiers given be 3000; their diſtance in ile 7 foot, and in Rank 3 foot;

The Proportion holds,

As 7 : to 3 : : ſo is 3000 : to 1286.

Extend the Compaſſes from 7, ownwards to 3, the ſame extent ill reach from 3000, downwards o 1286.

Seek 1286 in the firſt Radius of he Broken Line, and juſt againſt it you ſhall find 35,7, the number of men to be placed in File — 35 men is too little and 36 men will be too much; but men are not to be divided in parts.

Queſt. 5.

How to order any number of ſouldiers into Rank and File, ſo, that their diſtance in Rank ſhall be to the diſtance in File, in ſuch proportion as any two numbers given are?

IF 3000 ſouldiers were to be ordered in Rank and File, ſo that the diſtance between Rank and Rank ſhall be in proportion to the diſtance between File and File, as 5 is to 9 (that is) if the men in File ſtand 9 foot aſunder, the men in Rank ſhall ſtand 5 foot aſunder.

The Proportion is,

As 5 : to 9 : : ſo is 3000 : to 5400.

Extend the Compaſſes from 5 to

9, the ſame extent will reach from 3000 to 5400 — Seek 5400. in the Double Line of Numbers, and againſt it in the Broken Line, you ſhall find 73,5, for the number of men in Rank. — Then for the number of men in File,

As 73,5 : is to 1 :: ſo is 3000 to 41 *fere.*

Extend the Compaſſes from 73,5 to 1, the ſame extent will reach from 3000 to 41, and ſo many men muſt be in File — But here the number of men are 3013, which 13 over muſt be ſupplied, or elſe 28 men muſt be taken off and diſpoſed of as Scouts, Centinels, or the like; otherwiſe there muſt be one File leſs.

Queſt. 6.

There are 8100 in a ſquare Battail drawn up, and it is required to have 6 Ranks of Pikes to arme the ſame ſquare Body round about; how many Ranks muſt there be in the whole ſquare Battail, and what number of Pikes and what of Muskteers?

THe Square Root of 8100 is 90, the number of men in Rank and File; now for that there muſt be 6 Ranks of Pikes about the Muſketeers, there will be 12 Ranks leſs of them both in Front and Flank, than in the whole Body: wherefore ſubtract 12 from 90, there will remain 78, which number find in the Broken Line of Numbers, and right againſt it you ſhall find 6084, the number of Musketeers, and that taken from 8100, there remains 2061, for the number of Pikes.

SECT. II.

Concerning the Quartering of Souldiers by the Lines.

Queſt. I.

If 1000 *Souldiers may be lodged or quartered in a ſquare of* 300 *foot of Ground, how many foot long muſt the ſide of a ſquare be, that the Ground included may lodge* 5000 *?*

EXtend the Compaſſes from 1 to 300 (the ſide of the Square which will lodge 1000 Souldiers) the ſame extent will reach forward from 300 to 90000, then

The Proportion will be

As 1000 : is to 5000 :: ſo is 90000 : to 450000.

Seek this number in the Double Line of Numbers, and againſt it in the

the Broken Line you ſhall find 671, and ſo much muſt the ſide of a Square be, that muſt lodge 5000 Souldiers with the ſame convenience that 1000 Souldiers were lodged in a Square whoſe ſide was 300 foot.

According to this Method may all Queſtions of this kind be reſolved.

The

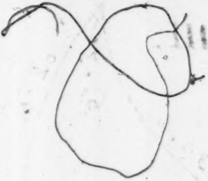


Fig: I.

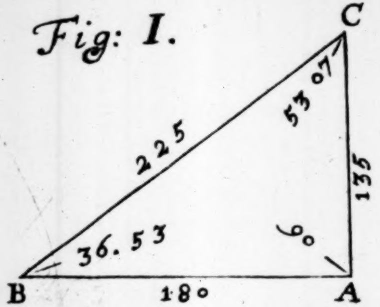

C
53 07
225
135
36. 53
60
B
180
A

Fig: III.

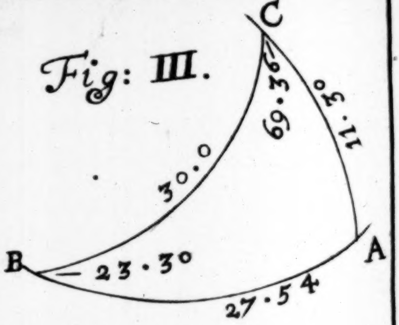

C
69.36
11.30
30.0
B
23.30
A
27.54

Fig. II.

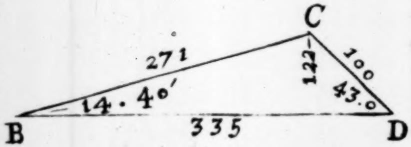
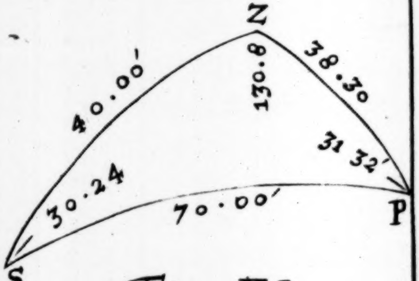

C
271
122
100
14.40′
43.0
B
335
D

Fig: IV.


Z
40.00′
130.8
38.30
31 32′
30.24
70.00′
P
S

Against page 67.

The USE of the
PROPORTIONAL
LINES
IN
TRIGONOMETRY:
OR,
The Menſuration of Triangles
BOTH
Plain and *Spherical.*

CHAP. V.

Definitions and Theorems Trigonometrical.

1 A Triangle is a Figure conſiſting of three Sides and as many Angles; as is the Triangle CAB, in *Fig.* I.

2. Any two Sides of a Triangle are called the Sides of the Angle contained by them; as the Sides CB and AB, are the Sides containing the Angle CBA.

3. The meaſure of an Angle is the quantity of the Arch of a Circle, deſcribed upon the angular Point, and cutting both the Sides containing the Angle.

4. A Degree is the 360 part of any Circle. Therefore,

5. A Semicircle contains 180 degrees. And

6. A Quadrant (or right Angle) contains 90 degrees.

7. The Complement of an Angle leſs than 90 degrees, is ſo much as that Angle wanteth of 90 degrees.

8. The Complement of an Angle to a Semicircle, is ſo much as that Angle wanteth of 180 degrees.

9. An Angle is either Right, Acute, or Obtuſe.

10. A Right Angle is that whoſe mea-

eaſure is 90 degrees, or a Qua-rant.

11. An Acute Angle is leſs than a ght Angle, and alwayes contains ſs than 90 degrees.

12. An Obtuſe Angle is greater han a right Angle, and alwayes ontains more than 90 degrees.

13. A Triangle is either right-ngled or oblique-angled.

14. A right-angled Triangle is uch a Triangle as hath one right ngle. As the Triangle C A B, (*Fig. I.*) hath one right Angle, amely, that at A, which contain-th juſt 90 degrees.

15. In every right-angled Trian-le. that Side which ſubtendeth (or ieth oppoſite to) the right Angle is alled the *Hypotenuſe*; and of the o-her two Sides, the one is called the *Perpendicular*, and the other the *Baſe*, at pleaſure: But moſt com-monly the ſhorter ſide is called the Perpendicular, and the longer the

Baſe. Thus in the Triangle C B A B C is the Hypotenuſe, C A the Perpendicular, and A B the Baſe.

16. In every right-angled Triangle, if you have one of the acu Angles given, the other is alſo give it being the Complement thereof t 90 degr. As in the Triangle C A B if you have the Angle at C 53 degr 7 min. given, you have alſo the Angle at B given, it being the Complement of that at C to 90 degr. wherefore take 53 degr. 7 min. from 9 degrees, and there will remain 3 degr. 53 min, which is the quantity of the Angle at B.

17. In all right-lined Triangle whatſoever (either right-angled o oblique-angled) the three Angles together are equal to two right Angles or contain 180 degrees: Therefore if you have any two Angles of Triangle given, you have alſo th third given, it being the Complement of the other two to 180 degrees;

grees;

rees: Thus, in the Triangle CDB,
ig. II. if there were given the Angle
DB, 43 deg. 20 min. and the Angle
BD 14 degrees 40 min. I ſay, by
onſequence you have the third An-
le DCB alſo given, it being the
Complement of the other two to
80 degr. For the two given An-
les BDC 43 degr. 20 min. and
BD 14 degr. 40 min. being added
ogether, make 58 degr. which be-
ng taken from 180 degr. there will
emain 122 deg. the quantity of
he obtuſe Angle DCB.

18. In all Triangles whatſoever,
he Sides are in proportion one to
he other as the Sines of the An-
les oppoſite to thoſe Sides. So in
he Triangle CDB, the Sine of the
Angle at D, is in proportion to the
Side CB, which is oppoſite to it,
as the Sine of the Angle at B, is to
he Side CD, or the Angle at C, to
he Side DB.

Theſe

[illegible] being premiſed, [illegible] ne Solution of *Pla* *Triangles* both *Right* and *Obliq* angled.

I. *Of Right angled Plain Triangles.*

THE Triangle which I ſhall mak uſe of in the ſeveral Caſes belonging to a *Right-angled Plain Triangle*, ſhall be that *Fig. I.* noted wit C A B, In which

		parts
A B the Baſe,	contain	180 .
C A the Perpendicular,		135
C B the Hypotenuſe,		225
And		deg.
A the Right Angle,	contains	90 —
C the Angle at the Per.		53 —
B the Angle at the Baſe,		36 —

CASE I.

The Baſe B A 180, *and the Perpendicular C A* 135, *being given,*
find the Angles B and C.

Th

The Proportion is,

As the Logarithm of A B
Is to the Logarithm of A C,
So is the Radius,
To the Tangent of B.

Extend the Compaſſes from 180 e Baſe, to 135 the Perpendicular, pon *the* Line of Numbers, the ſame xtent will reach, the ſame way, om the Radius (or Tangent of 45 g.) to the Tangent of 36 deg. 3 min. the quantity of the Angle t B.

CASE II.

e Hypotenuſe C B 225, and the Baſe A B 180, being given, to find the Angles B and C.

The Proportion is,

As the Logarithme of C B,
Is to the Radius;

So is the Logarith. of the Side AB
To the Sine of C.

Extend the Compaſſes from 22 the Hypotenuſe, to the Radius (o Sine of 90 degr.) the ſame exte will reach, the ſame way, from 18 the Baſe, to 53 deg. 7 min. the quantity of the Angle at C.

Or,

The diſtance between 225 a 180, will reach from the Sine of 9 to the Sine of 53 degr. 7 min. a before.

CASE III.

The Baſe AB 180, the Angle C 5 degr. 7 min. and the Angle B 3 deg. 53 min. being given, to find t Perpendicular C A.

The Proportion is,

As the Sine of the Angle at C,
Is to the Logar. of AB,
So is the Sine of the Angle B,
To the Logar of C A.

Or,

As the Radius,
Is to the Logar. of AB,
So is the Tangent of B,
To the Logar. of C A.

Extend the Compaſſes from the ine of 53 deg. 7 min. the Angle at , to 180 the Baſe, the ſame ex- nt will reach from the Sine of 6 degr. 53 min. to 135 the Perpen- icular C A.

Or,

Extend the Compaſſes from the angent of 45 deg. to 180 the Baſe, e ſame extent will reach, the ſame ay, from the Tangent of 36 deg. 3 min. to 135 the Perpendicular, s before.

CASE IV.

he Hypotenuſe CB 225, the Angle C 53 deg. 7 min. and the Angle at B 36 deg. 53 min. given, to find the Baſe B A, and the Perpendicular C A.

The Proportion is,

As the Radius,
Is to the Logar. of CB,
So is the Sine of C,
To the Logar. of AB.
And the Sine of B,
To the Logar. of CA.

Extend the Compaſſes from th Sine of 90, to 225 the Hypotenuſe; the ſame extent will reach from th Sine of 53 deg. 7 min. the Angl at C, to 180 the Baſe AB — And likewiſe, the ſame extent wil reach from the Sine of 36 deg 53 min. to 135, the Perpendicu lar CA.

CASE V.

The Hypotenuſe CB 225, and the Ba AB 180, being given, to find th Perpendicular CA.

The Proportion is,

1. *Operation.*

As the Logar. of C B,
Is to the Radius;
So is the Logar. of A B,
To the Sine of C.

2. *Operation.*

As the Radius,
Is to the Logarithm of C B,
So is the Sine of B (the Complement of C)
To the Logar. of C A.

Extend the Compaſſes from 225 he Hypotenuſe, to the Sine of 90, e ſame extent will reach from 180 e Baſe, to the Sine of 53 degr. min. the Angle at C.

Again,

Extend the Compaſſes from the ine of 90, to 225 the Hypotenuſe, e ſame extent will reach from the ne of 36 degrees 53 minutes, the

So is Tangent Z P 38 d. 30 m.
To Tangent P R 34 d. 6 m.

2. *Operation.*

As Co-ſine P R 55 d. 54 m.
Is to Co-ſine S R 54 d. 6 m.
So is Co-ſine Z P 51 d. 30 m.
To Co-ſine Z S 50 d.

I.

Extend the Compaſſes from the Sine of 90 deg. to the Co-ſine of P 58 deg. 28 min. the ſame extent will reach from the Tangent of Z P 38 deg. 30 min. to the Tangent of 34 deg. 6 min. for P R.

II.

Extend the Compaſſes from the Co-ſine of P R 55 deg. 54 min. to the Co-ſine of S R 54 deg. 6 min. the ſame extent will reach from the Co-ſine of Z P 51 deg. 30 min. to the Co-ſine of 50 deg. for Z S.

CASE V.

Two Sides ZP *and* SP, *with the Angle* P *contained between them, given, to find the Angle* S, *oppoſite to the Angle* P.

The Proportion is,

1. *Operation.*

As Radius 90 d.
Is to Co-ſine P 58 d. 28 m.
So is Tangent ZP 38 d. 30 m.
To Tangent RP 34 d. 6 m.

2. *Operation.*

As Sine PR 34 d. 6 m.
Is to Sine SR 35 d. 54 m.
So is Tangent P 31 d. 32 m.
To Tangent S 30 d. 24 m.

I.

Extend the Compaſſes from Radius 90 deg. to the Co-ſine of P 31 deg. 32 min. the ſame extent will reach from the Tangent of ZP 38 deg. 30 min. to the Tangent of 30 deg. 24 min. for the Angle at S.

II.

Extend the Compaſſes from the Sine of P R 34 d. 6 m. to the Sine of S R 35 d. 54 m. the ſame extent will reach from the Tangent P 31 deg. 32 m. to the Tangent of 30 deg. 24 m. for the Angle at S.

CASE VI.

Two Sides Z P *and* Z S, *with the Angle* P *oppoſite to* S Z, *given*, *to find the Angle* Z, *contained between the two given Sides.*

IN this Caſe the Baſe is alwayes the Side unknown.

The Proportion is,
1. *Operation.*

As Radius 90 d.
Is to Co-ſine Z P 38 d. 30 m.

So is Tangent P 31 d. 32 m.
To Tangent R Z P 25 d. 38 m.

2. *Operation.*

As Tangent Z S 40 d.
Is to Tangent Z P 38 d. 30 m.
So is Co-ſine R Z P 64 d. 22 m.
To Co-ſine S Z R 58 d. 44 m.

I.

Extend the Compaſſes from Radius 90 deg. to the Co-ſine of Z P 51 deg. 30 min. the ſame extent will reach from the Tangent of P 31 deg. 32 min. to the Tangent of 25 deg. 38 min. for the Angle R Z P.

II.

Extend the Compaſſes from the Tangent of Z S 40 deg. to the Tangent of Z P 38 deg. 30 min. the ſame extent will reach from the Co-ſine of R Z P 64 deg. 22 min. to the Co-ſine of 31 deg. 16 min.

CASE VII.

Two ſides Z S and Z P, with the angle S oppoſite to Z P given, to find the ſide S P adjacent to the given angle S.

The Proportion is,

1. *Operation.*

As Radius 90 d.
Is to co-ſine S 59 d. 36 m.
So is tangent Z S 40 d.
To tangent S R 35 d. 54 m.

2. *Operation.*

As co-ſine S Z 50 d.
Is to co-ſine Z P 51 d. 30 m.
So is co-ſine S R 54 d. 6 m.
To co-ſine P R 55 d. 54 m.

I.

Extend the Compaſſes from Radius 90 deg. to the co-ſine of S 59 d. 36 m. the ſame extent will reach from the tangent of Z S 40 deg. to the tangent of S R 35 deg. 54 min.

II.

II.

Extend the Compaſſes from the co-ſine of SZ 50 deg. to the coſine of ZP 51 deg. 30 min. the ſame extent will reach from the co-ſine of SR 54 deg. 6 min. to the co-ſine of PR 55 deg. 54 min.

CASE VIII.

Two angles S and Z, with the ſide SZ included between them given, to find the angle P oppoſite to the given ſide SZ.

IN this Caſe the Baſe may be either of the unknown ſides.

The Proportion is,
1. *Operation.*

As Radius 90 d.
Is to co-ſine *S*Z 50 d.
So is tangent *S* 30 d. 24 m.
To co-tangent RZ*S*.

2. *Operation.*

2. *Operation.*

As ſine R Z S
To ſine R Z P;
So is co-ſine S 59 d. 36 m.
To co-ſine P 58 d. 28 m.

I.

Extend the Compaſſes from Radius 90 deg. to the co-ſine of S Z 50 deg. the ſame extent will reach from the tangent of S 30 degrees 24 minutes, to the co-tangent of R Z S.

II.

Extend the Compaſſes from the ſine of R Z S to the ſine of R Z P, the ſame extent will reach from the co-ſine of S 59 degrees 36 minutes, to the co-ſine of P 58 degrees 28 minutes.

CASE IX.

Two angles Z and P, with the ſide ZP between them, given, to find the ſide ZS oppoſite to the given angle at P.

IN this Caſe the Baſe is the ſide neither given nor ſought as SP.

The Proportion is,

1 *Operation.*

As Radius 90 d.
  Is to co-ſine **ZP** 51 d. 30 m.
So is tangent **P** 31 d. 32 m.
  To co-tangeut **RZP.**

2. *Operation.*

As co-ſine **RZ***S*
  Is to co-ſine **RZP,**
So is tangent **ZP**
  To tangent **Z***S*

I.

Extend the Compaſſes from Radius 90 degr. to the co-ſine of **ZP** 51

51 degr. 30 min. the ſame extent will reach from the tangent of P 31 degr. 32 min. to the co-tangent of RZP.

II.

Extend the Compaſſes from the co-ſine of RZS, to the co-ſine of RZP, the ſame extent will reach from the tangent of ZP 38 deg. 30 min. to the tangent of ZS 40 degrees.

CASE X.

Two angles S and P, with a ſide oppoſite to one of them SZ, *given*, *to find the other angle Z.*

IN this Caſe the Baſe is the ſide oppoſite to the angle ſought.

The Proportion is,
1. *Operation.*

As Radius 90 d.
Is to co-ſine ZS 50 d.

So is tangent S 30 d. 24 m.
To co-tangent SZR.

2. *Operation.*

As co-ſine S 59 d. 36 m.
Is to co-ſine P 58 d. 28 m.
So is ſine SZR,
To ſine RZP.

I.

Extend the Compaſſes from Radius 90 degr. to the co-ſine of ZS 30 degr. the ſame extent will reach from the tangent of S 30 degr. 24 min. to the co-tangent of SZR.

II.

Extend the Compaſſes from the co-ſine of S 59 degr. 36 min. to the co-ſine of P 58 d. 28 m. the ſame extent will reach from the ſine of SZR to the ſine of RZP.

CASE XI.

The three ſides S Z, P Z, and S P, given, to find an angle, viz. the angle at Z.

IN this Caſe the ſide oppoſite to the inquired angle is the Baſe.

Before the Triangle can be reſolved, you muſt

Firſt, Adde the three ſides together, and note the ſum of them.

Secondly, Take the half thereof, which call the half ſum.

Thirdly, From the half ſum, ſubſtract the Baſe, and note the difference, as you ſee here done.

The Side	SZ	40	00
	ZP	38	30
	SP	70	00
The Sum		148	30
The half Sum		74	15
From which ſubſtract the Baſe 70 deg there remains the difference		4	15

This preparation being made, the proportion will be,

1 *Operation.*

As Radius 90 d.
Is to ſine ZS 40 d.
So is the ſine of ZP 38 d. 30 m.
To a fourth ſine, *viz.* 23 d. 35 m.

2 *Operation.*

As the ſine of 23 d. 35 m.
Is to the ſine of the half ſum 74 d. 15 m.
So is the ſine of the difference 4 d. 15 m.
To a ſeventh ſine, *viz.* 10 d. 17 m.

I.

Extend the Compaſſes from Radius 90 deg. to the ſine of ZS 40 deg. the ſame extent will reach from the ſine of ZP 38 deg. 30 min. to a fourth ſine, *viz.* 23 deg. 35 min.

II.

Extend the Compaſſes from the ſine of 23 deg. 35 min. to the ſine of the half ſum 74 degr. 15 min. t ſame extent will reach from the ſi of the difference 4 deg. 15 min. to ſeventh ſine, *viz.* 10 deg. 17 min.

Divide the ſpace upon the Line o Sines betwen 10 deg 17 min. and 90 degr. into two equal parts, and the Compaſs point ſhall reſt upo 24 deg. 56 min. whoſe Complemen is 65 deg. 4 min. and that double makes 130 deg. 8 min. for the angle at Z.

CHAP. XII.

The three angles Z, S, and P, given, t find a ſide.

THis is but the converſe of th former Caſe, and may be reſolved in the ſame manner, if for either of the angles next to the ſide required, you take its complement t 180 deg, thoſe angles will be turne into ſides, and the ſides into angles and then may the triangle be reſolve as in the preceding Caſe.

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Fig: 1.

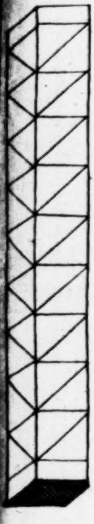


Fig: 2.

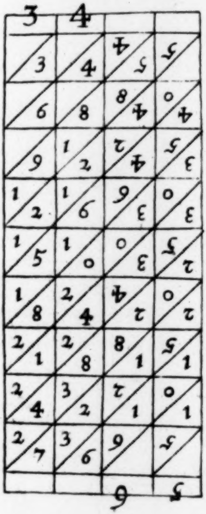
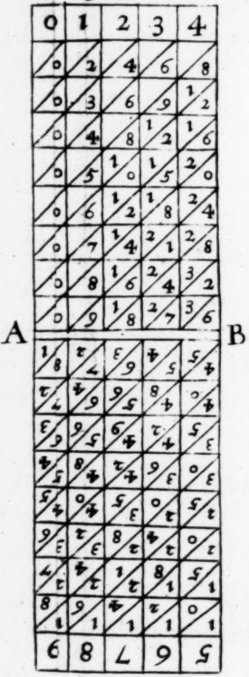
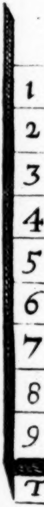


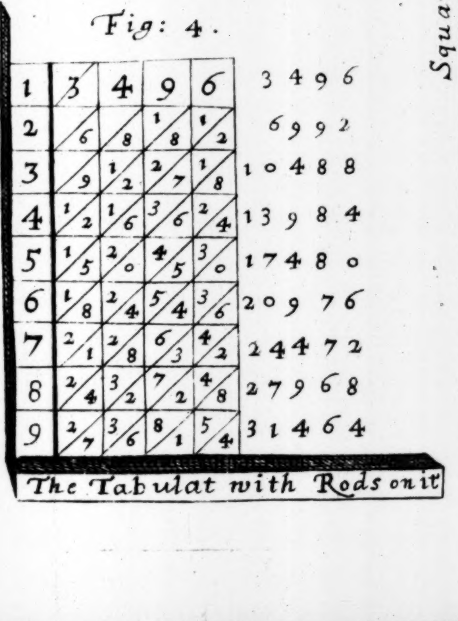
Fig: 3.




1
2
3
4
5
6
7
8
9
T

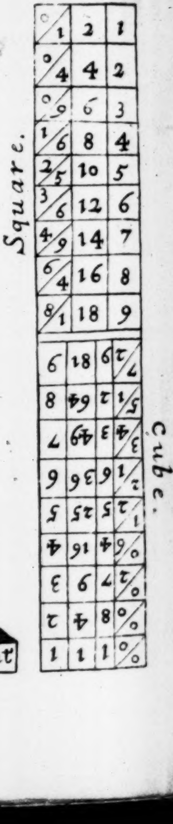
Place this Againſt. 121. part third to fold out.

Fig: 4.



1	3	4	9	6	3496
2	6	8	1/8	1/2	6992
3	9	1/2	2/7	1/8	10488
4	1/2	1/6	3/6	2/4	13984
5	1/5	2/0	4/5	3/0	17480
6	1/8	2/4	5/4	3/6	20976
7	2/1	2/8	6/3	4/2	24472
8	2/4	3/2	7/2	4/8	27968
9	2/7	3/6	8/1	5/4	31464

The Tabulat with Rods on it



Square.

0/1	2	1
0/4	4	2
0/9	6	3
1/6	8	4
2/5	10	5
3/6	12	6
4/9	14	7
6/4	16	8
8/1	18	9

Cube.

0/0	1	1	1
0/0	8	4	2
0/2	7	9	3
0/6	4	16	4
1/2	5	25	5
2/1	6	36	6
3/4	3	49	7
5/1	2	64	8
7/2	9	81	9

inſt. 121. part third

old out.

4.

6	3496
1/2	6992
1/8	10488
2/4	13984
3/0	17480
3/6	20976
4/2	24472
4/8	27968
5/4	31464

t with Rods on it

Square.

0/1	2	1
0/4	4	2
0/9	6	3
1/6	8	4
2/5	10	5
3/6	12	6
4/9	14	7
6/4	16	8
8/1	18	9

Cube.

0/0	1	1	1
0/0	8	4	2
0/2	7	9	3
0/6	4	16	4
1/2	5	25	5
2/1	6	36	6
3/4	3	49	7
5/1	2	64	8
7/2	9	81	9

The USE of the PROPORTIONAL LINES IN *ASTRONOMY*.

CHAP. V.

Argument.

I Shall not in this place go about to give you any Deſcription of the Circles of the Sphere or Globe, ſuppoſing my Reader to be acquainted with them already; and in reſpect I have ſufficiently treated of them elſewhere, as in my *Uſes of the Globes*, and alſo in my *Geometrical Exerciſes*; which Book will explain and make eaſie ſome things which in this Tractate may be omitted, or at leaſt, for brevity, lightly paſſed over.

Probl. I.

The diſtance of the Sun *from the near-eſt* Æquinoctial Point (*either* Aries *or* Libra) 59 *deg. given, to fin* *his* Declination.

The Proportion is,

As the Radius 90 deg.
Is to the Sine of the Sun's greate
Declination 23 deg. 30 m.
So is the Sine of the Sun's diſtan
from the next Æquinoctial Poi
Libra 59 degr.
To the ſine of the Sun's preſe
Declination 20 degr.

Extend the Compaſſes from t ſine of 90, to the ſine of 23 deg. min. (the *Suns* greateſt Declin on) the ſame extent will reach fr 59 deg. (the *Suns* diſtance fr

Li

Libra, to the ſine of 20 deg. the Suns preſent Declination.

The like Declination *the* Sun *hath when he is in* 29 *degr. of* Taurus, *in* 1 *degr. of* Leo, *or* 29 *degr. of* Scorpio, *every of which Points are diſtant from one of the* Æquinoctial Points Aries *or* Libra 59 *deg.*

Probl. II.

The Latitude *of the Place*, 51 *deg.* 30 *min. and the* Declination *of the* Sun 20 *deg. being given, to find the* Aſcenſional Difference.

The Proportion is,

As the co-tangent of the Latitude 38 deg. 30 min.
Is to the tangent of the Suns Declination 20 degr.
So is the Radius 90 deg.
To the ſine of the Aſcenſional Difference 27 deg. 14 min.

Extend the Compaſſes from the tangent of 38 deg. 30 min. the complement of the Latitude, to 20 deg. (the Suns Declination) the ſame extent will reach, the ſame way, from the ſine of 90 deg. to the ſine of 27 deg. 14 min. the Aſcenſional difference; which is the quantity of time that the Sun riſes or ſets before or after Six of the Clock.

So theſe 27 degr. 14 min. being turned into Time (by allowing 15 deg. for one hour, and one degree for 4 minutes of Time) is 1 hour and 49 min. and ſo much doth the Sun riſe or ſet before or after the hour of Six, according to the time or ſeaſon of the year; for if the Sun hath *North Declination*, then he *riſeth before ſix* and *ſets after:* but if the Sun have *South Declination*, then doth he *riſe after*, and *ſets before Six.*

This Aſcenſional Difference *being added to Six hours, will give you the*

Semidiurnal Arch *or Half-length of the Day; and being taken from Six hours, will leave the* Seminocturnal Arch, *or Half-length of the Night.*

Probl. III.

The Latitude *of the Place* 51 *deg.* 30 *min. and the* Declination *of the Sun*, 20 *deg. being given, to find his* Amplitude.

The Proportion is,

As the co-ſine of the Latitude 38 deg. 30 min.
Is to the Radius 90 deg.
So is the ſine of the Sun's Declination 20 degr.
To the ſine of the Amplitude from the Eaſt or Weſt points of the Horizon 33 degr. 20 min.

Extend the Compaſſes from the ſine of 38 deg. 30 min (the Complement of the Latitude) to the ſine of 90 deg. the ſame extent will reach from the ſine of 20 deg. (the Suns Declination) to 33 deg. 20 min. (the Amplitude, or) the diſtance that the Sun riſes or ſets from the true Eaſt or Weſt Points, towards either the North or South.

Probl. IV.

The Latitude *of the Place*, 51 *deg*, 30 *min. and the* Declination *of the* *Sun* 20 *deg. being given, to find the* Angle of the Sun's Poſition *at the time of his riſing.*

The Proportion is,

As the co-ſine of the Declination 70 degr.

Is to the Radius 90 degr.

So is the ſine of the latitude 51 degr. 30 min.

To the ſine of the Angle of the Suns Poſition at the time of his riſing.

Extend the Compaſſes from the ſine of 70 deg. (the complement of the Suns Declination) to the ſine of 90; the ſame extent will reach from the ſine of 51 deg. 30 min. the latitude) to the ſine of 56 deg. 29 min. (the angle of the Sun's poſition at the time of his riſing.)

Probl. V.

The Sun's Declination 20 *deg. and his* Amplitude 33 *deg.* 20 *min. from the* Eaſt *or* Weſt *part of the* Horizon, *being given, to find the* Latitude.

The Proportion is,

As the ſine of the Amplitude from the Eaſt or Weſt 33 deg. 20 min.

Is to the Radius 90 deg.

So is the ſine of the Declination 20 deg.

To the co-ſine of the Latitude 38 degr. 30 min.

Extend the Compaſſes from the ſine of 33 deg. 20 min (the Sun's Amplitude from the Eaſt or Weſt) to the ſine of 90 deg. the ſame extent will reach from the ſine of 20 deg. (the Sun's Declination) to the ſine of 38 deg. 30 min. (the complement of the Latitude, 51 deg. 30 min.)

Probl. VI.

The Sun's greateſt Declination 23 *deg.* 30 *min. with his Diſtance from the next* Æquinoctial Point (Aries *or* Libra, 59 *deg.*) *being given, to find his* Right Aſcenſion.

The Proportion is,

As the Radius 90 deg.
Is to the co-ſine of the greateſt Declination 66 deg. 30 min.
So is the tangent of the Sun's diſtance from the next Æquinoctial point *Libra* 59 deg.
To the tangent of the Right Aſcenſion 56 deg. 50 min.

Extend the Compaſſes from the

ſine of 90 deg. to the ſine of 66 deg. 30 min. (the complement of the Sun's greateſt Declination;) the ſame extent will reach from the tangent of 59 deg. (the Suns diſtance from the next Æquinoctial Point) to the tangent of 56 deg. 50 min. (the Suns Right Aſcenſion.)

Probl. VII.

The Latitude *of the Place* 51 *deg.* 30 *min. and the Suns* Declination 20 *deg. being given, to find at what hour the Sun will be upon the true* Eaſt *or* Weſt *Points.*

The Proportion is,

As the tangent of the Latitude 51 deg. 30 min.
Is to the tangent of the Suns Declination 20 degr.
So is the Radius 90 degr.
To the co-ſine of the Hour from Noon,

Extend the Compaſſes from the tangent of 51 deg. 30 min (the Latitude) to the tangent of 20 deg. (the Suns Declination) the ſame extent will reach from the ſine of 90 deg. to the ſine of 16 deg. 50 min. the complement of the time from Noon, that the Sun will be due Eaſt or Weſt.

Which converted into hours and minutes, will be 4 hours and about 53 min. So that the Sun, when he hath 20 degr. of Declination, will come to the Eaſt Point at 7 min. paſt 7 in the Morning, and will be due Weſt 53 min. after 4 in the Afternoon.

Probl. VIII.

Having the Latitude of the Place 51 deg. 30 min. and the Suns Declination 20 *deg. given, to find what* Altitude *the Sun ſhall have when he is upon the true* Eaſt *or* Weſt *Points.*

The Proportion is,

As the ſine of the Latitude 51 deg. 30 min.

Is to the Sine of the Declination 20 degr.

So is the Radius 90 degr.

To the Sine of the Suns Altitude being due Eaſt or Weſt 25 degr. 55 min.

Extend the Compaſſes from the ſine of 51 deg. 30 min. (the Latitude) to the ſine of 20 deg. (the Declination) the ſame extent will reach from the ſine of 90 deg. to the ſine 25 deg. 55 min. the Altitude that the Sun ſhall have when he is upon the Eaſt or Weſt Points.

Probl. IX.

The Latitude *of the Place* 51 *deg.* 30 *min. and the Suns* Declination 20 *deg. being given, to find what* Altitude *the Sun ſhall have at Six of the Clock.*

The Proportion is,

As the Radius 90 degr.
Is to the ſine of the Suns Declination 20 degr.
So is the ſine of the Latitude 51 deg. 30 min.
To the ſine of the Suns Altitude at Six, 15 deg. 30 min.

Extend the Compaſſes from the ſine of 90 deg. to the ſine of 20 deg. (the Suns Declination) the ſame extent will reach from the ſine of 51 deg. 30 min. (the Latitude) to the ſine of 15 deg. 30 min. (the Altitude that the Sun ſhall have at Six of the Clock.)

Probl. X.

The Latitude *of the Place* 51 *deg.* 30 *min. and the* Declination *of the Sun* 20 *deg. being given, to find what* Azimuth *the Sun ſhall have at Six a Clock.*

The Proportion is,

As the co-ſine of the Latitude 38 degr. 30 min.

Is to the Radius 90 deg.

So is the co-tangent of the Suns Declination 70 degr.

To the tangent of the Suns Azimuth counted from the North part of the Meridian 77 deg. 14 min.

Extend the Compaſſes from the ſine of 38 deg. 30 min (the complement of the Latitude) to the ſine of 90 deg. the ſame extent will reach from the tangent of 70 deg. (the complement of the Suns Declination) to the tangent of 77 deg. 14 min.) the Suns Azimuth counted from the North part of the Meridian) or 12 degr. 46 min. the Azimuth from the Eaſt or Weſt, or 102 deg. 46 min. from the South.

Probl.

Probl. XI.

The Latitude *of the Place* 51 *deg.* 30 *min. the* Declination *of the Sun*, 20 *deg.* South, *and the Suns* Altitude 12 *deg. given*, *to find the Suns* Azimuth *either from the* Eaſt, North, *or* South *Points of the* Horizon.

TO reſolve this Problem, you muſt find the Complement of the Latitude, the Complement of the Altitude, and the Complement of the Declination, and add all three of them into one Snm, and take the half thereof; from which half ſum ſubſtract the Complement of the Suns Declination, and note the difference; as you ſee here done.

Com-

Complement of the	Latitude	38	30
	Altitude	78	00
	Declinat.	110	00
	Their Sum	226	30
	Half Sum	113	15
	Comp. Declinat. ſub.	110	00
	The Difference	3	15

Having found the Sum, the half Sum, and the Difference, you may work by the following

Proportion,

1. As the Radius 90 degr.
Is to the co-ſine of the Latitude, 38 deg. 30 min.
So is the co-ſine of the Altitude 78 degr.
To the ſine of a fourth number; which is 37 deg. 30 min.

2. As

Take this fourth Term 16 deg. 1 min. from 57 deg. 35 min. the complement of the leſſer Latitude, and the remainer will be 41 deg. 34 min.

And ſay again,

(2.) As the ſine of 73 deg. 59 min. (the complement of the fourth Term before found)

To 48 deg. 26 min. (the complement of the Remainer,)

So is the ſine of 50 deg. (the greater Latitude,)

To the ſine of 36 deg. 36 min. (whoſe complement 53 deg. 24 min.) is the diſtance, which in miles is 3205.

The

The USE of the PROPORTIONAL LINES IN NAVIGATION.

CHAP. VIII.

THe principal Problems in uſe with Mariners in their Navigations (beſides thoſe of Aſtronomy and Geography in the foregoing Chapters) are ſuch as concern *Longitude*, *Latitude*, *Rumb*, and *Diſtance*, a few of which I ſhall ſhew how to perform by the *Proportional Lines*.

Examples in Figure I.

In which Figure,

C A. repreſent the Meridian, *C* North and *A* South.

B A, A Parallel of Latitude, *B* Weſt and *A* Eaſt.

C B, A Rumb 53 deg. 7 min. diſtant from the Meridian Weſtward, which Rumb is N. W. 8 deg. 7 min. Weſterly from *C*. and N. E. by N. 19 min. Eaſterly.

And ſo

C B Is the Courſe or Rumb.

C A The difference of Latitude, and

B A The departure from your firſt Meridian.

Probl. 1.

Probl. I.

The courſe and diſtance given, to find the difference of Latitude and departure from your firſt Meridian.

SAiling from C 225 min the Courſe or Rumb is N W 8 deg. 7 min. Weſterly (that is 53 deg. 7 min. from the Meridian) I demand how much I have altered my Latitude, and how far I have departed from my firſt Meridian.

The Proportion is,

As Radius 90 deg.

Is to the diſtance ſailed 225 min.

So is the ſine of the Rumb 53 deg. 7 min.

To 180 the departure from your firſt Meridian.

And

So is the complement of the Rumb 36 deg. 53 min.

To 135 min. the difference of Latitude 1.

Extend the Compaſſes from the ſine of 90 deg. to 225, the ſame extent will reach from 53 deg. 7 min. the Rumb, to 180 min. for your departure: —— And alſo the ſame extent will reach from 36 deg. 53 min. the complement of the Rumb, to 135 min. for the difference of Latitude.

Probl. 2.

The courſe and difference of Latitude given, to find the diſtance ſailed, and the departure from your firſt Meridian.

Let the Courſe be N. W 8 deg. 7 min. Weſterly (or 53 deg. 7 min. from the Meridian) as before; the difference of Latitude 135 min. and let the diſtance ſailed CB, and the departure B A be required.

The Proportion is,

As the Co-ſine of the Courſe 36 deg. 53 min.
Is to 135 min. the difference of Latitude;
So is Radius 90 deg.
To 225 min. the diſtance ſailed.

And

So is the ſine of the Rumb 53 deg. 7 min.
To 180 min. the departure.

Extend the Compaſſes from 36 deg. 53 min. to 135, the ſame extent will reach from 90 deg. to 225 for the diſtance ſailed: — And from 53 deg. 7 min. to 180 min. the departure from your Meridian.

Probl. 3.

Probl. 3.

The courſe and departure being given, to find the diſtance ſailed and the difference of Latitude.

LEt the courſe be N. W. 8 deg. 7 min. Weſterly (or 53 deg. 7 min. from the Meridian) and the departure from the Meridian 180 min. and let the diſtance ſailed and the difference of Latitude be required.

The proportion is,

As the ſine of the courſe 53 deg. 7 min.
Is to the departure 180 min.
So is Radius 90 degr.
To 225 the diſtance ſailed.

And

So is the complement of the courſ 36 deg. 53 min.
To 135 min. the difference o Latitude.

Exten

Extend the Compaſſes from the ſine of 53 deg. 7 min. to 180 min. he ſame extent will reach from the ſine of 90 deg. to 225 min. the diſtance ſailed; and the ſame extent alſo will reach from 36 deg. 53 min. the complement of the courſ,to 135, min. the difference of Latitude.

Probl. 4.

The difference of Latitude and diſtance ſailed, given, to find the courſe and departure from the Meridian.

A Ship ſails between the North and the Weſt 225 min. ſo long till ſhe hath altered her Latitude 135 min. I demand what courſe the Ship hath made, and alſo how far ſhe hath departed from her firſt Meridian.

The Proportion is,

As the ſine of 90 degr.
Is to 225 m. the diſtance ſailed,

So is 135 min. the difference of Latitude,

To 36 deg. 53 min. the complement of the courſe that the Ship ſailed.

And

So is the ſine of 53 deg. 7 min.

To 180 min. the departure.

Extend the Compaſſes from 225 min. to the ſine of 90 deg. the ſame extent will reach from 135 min. to 36 deg. 53 min. whoſe complement 53 deg. 7 min. is the courſe. — And the ſame extent alſo will reach from 53 deg. 7 min. to 180 min. the Ships departure from the firſt Meridian.

Probl. 4.

Probl. 4.

The diſtance and departure given, to find the courſe and difference of Latitude.

THe diſtance ſailed is 225 min. and the departure is 180 min. I demand the courſe and difference of the Latitude: For which

The proportion is,

As 225 the diſtance ſailed,
Is to the ſine of 90 deg.
So is 180 the departure,
To the ſine of 53 deg. 7 min. the courſe.

And

So is the ſine of 36 deg. 53 min. the Complement of the Courſe,
To 135 min. the difference of Latitude.

Extend the Compaſſes from 225 min. the diſtance, to 90 deg. the ſame extent will reach from 180 min. the difference, to 53 deg. 7 min. the courſe, which is N. W. 8 deg. 7 min. Weſterly —— And the ſame extent will reach from 36 deg. 53 min. the complement of the courſe, to 135 min. the difference of Latitude.

Probl. 5.

The diffeeence of Latitude and departure given, to find the courſe and diſtance.

THe difference of Latitude is 135 min. and the departure is 180 min. the Rumb and Diſtance is required:

The Proportion is,

As 135 min. the difference of Latitude,

Is to Radius (or Tangent of 45 degr.)

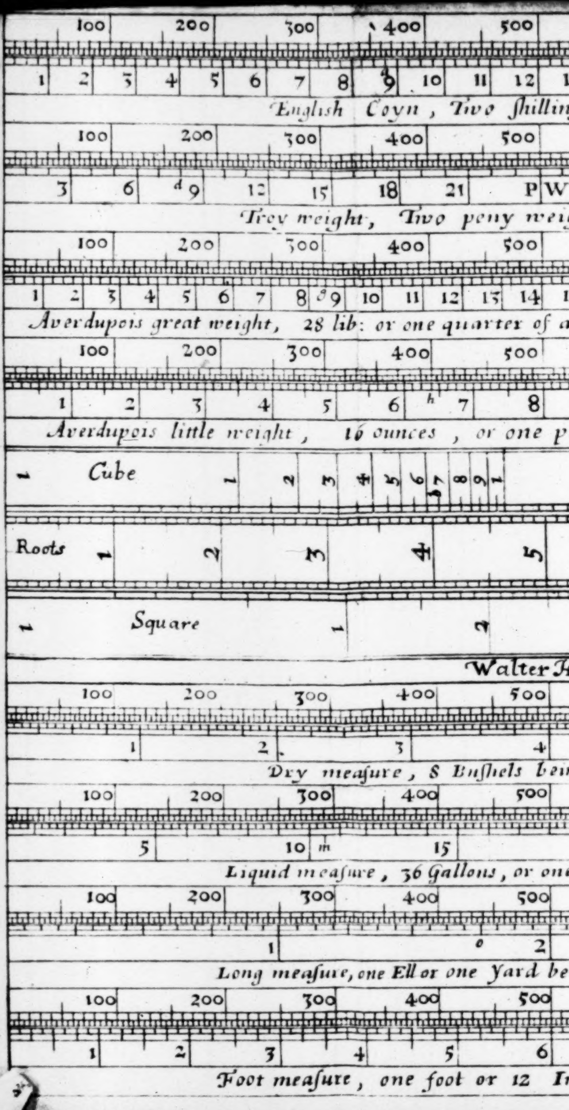
So is 180 min. the diſtance,
To the tangent of 53 deg. 7 min. the courſe.

And

So is the tangent of 45 deg.
To 225 min. the diſtance ſailed.

Extend the Compaſſes from 135 min. the difference of Latitude, to the tangent of 45 deg. the ſame extent will reach from 180 min. the diſtance, to the tangent of 53 deg. 7 *min.* or N. W. 8 deg. 7 min. Weſterly for the courſe. —— And the ſame extent alſo will reach from the tangent of 45 deg. to 225 min. diſtance ſailed.

FINIS.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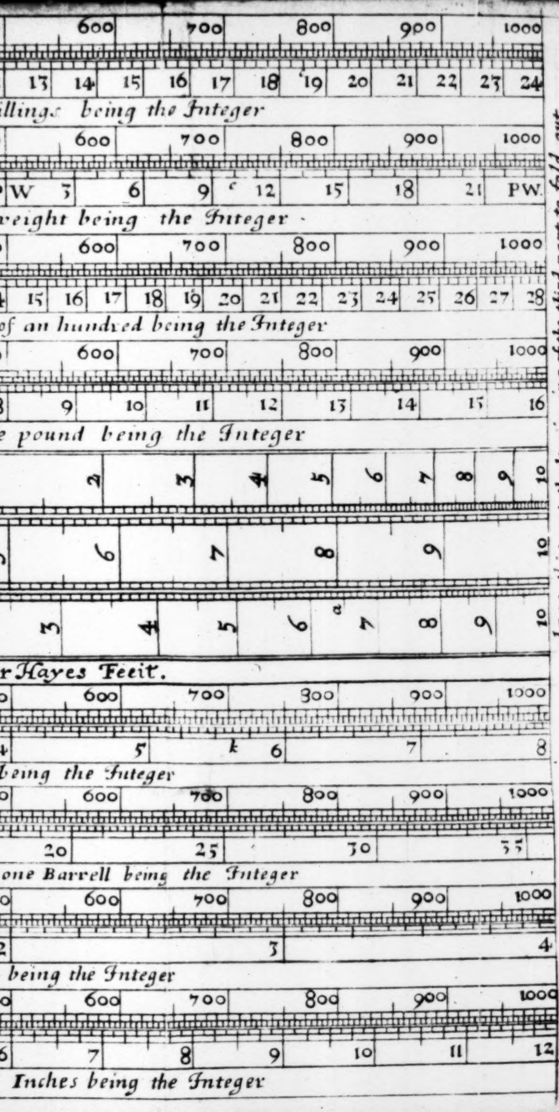
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6 7 8 9 10
3 4 5 6 7 8 9 10
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being the Integer
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20 25 30 35
one Barrell being the Integer
600 700 800 900 1000
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600 700 800 900 1000
7 8 9 10 11 12
Inches being the Integer

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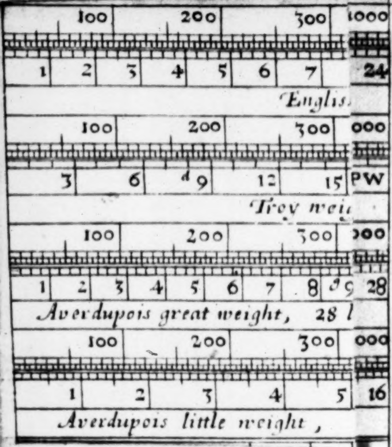
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beginning of the third part to fold out.

A
SUPPLEMENT
TO THE
LINE of PROPORTION,
OR,
NUMBERS.

Containing
The Deſcription, and ſome Uſes of
a convenient Two-Foot
JOYNT-RULE:
pon which is inſcribed divers LINES
and SCALES for ſeveral Uſes ſuta-
ble to all ſorts of Artificers, or Work-
mens occaſions.
nd alſo by the ſame RULE to take the
Heights and Diſtances, and to make
the moſt uſual ſorts of SUN-DIALS.

By *JOHN BROWN.*

LONDON:
Printed in the Year, 1676.

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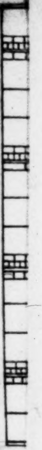
INTRODUCTION.

HAving formerly endeavoured, to promote the uſe of ſo excellent a Line, as that of the Line of Numbers, and finding by a ſeries of experience of my ſelf and others, that the greateſt difficulty lies in Reading thereof, and in the diſcovery of the right Number of Places, in Multiplication or Diviſion.

Therefore to make this difficulty eaſie and familiar, and certain for young beginners, or better proficients coming ſuddenly to the uſe thereof I have drawn this Line to 8 Radiuſſes, or 8 Revolutions of one Line of Numbers and added thereunto a Line of Time and Money, (and after



after the ſame manner may any other pa ticular thing be added, according to a mans occaſion) endeavouring thereb as much as may be, to make the Li ſpeak, as it were: Whoſe Application ſhort is thus.

The firſt 1 at the beginning repreſen the Ten Thouſandth part of an Integer or 1; the ſecond 1 is the Thouſandt part of any thing; the third 1 is th Hundredth part of an Integer; the fourt 1 is the Tenth part of any thing; an the fifth 1 at the end of the print is Integer or 1 of any thing; and next t this, above this Line of Numbers, is Line of Time, fitted for Queſtions of I tereſt at *6 per Cent. per Annum.*

And next to it, Under it is a Line fo Money, 1 Pound or 20 *s.* being Integer, giving the intereſt of a 100 for any time by Inſpection, and of an other Sum by one operation for any tim whatſoever.

In the middle of the Whit Blanck below this Line of Pence, is this print t

b

e cut, and 1 at the end of the firſt Lines, o be paſted exactly on 1 on the beginning of the other Lines; then the firſt. 1 here, repreſents 1 of any thing, as 1 Foot, 1 Inch, 1 Pound, &c. the ſecond 1 repreſents 10 Integers, the third 100 Integers, the fourth 1000 Integers, the fifth 10000 Integers, &c. And the Figures and Cuts beween every Radius or 1 expreſſed as much as may be, and being ſo Cut and Paſted together, you may read any Number from the 1000 part of an Integer, o 10000 Integers, and 1 being in the middle, is convenient to extend to any Numbers under or over 1; and thereby Fractions in Money or Time are wrought s ſoon as whole Numbers, and as eaſily, s by a little practice you ſhall find; and f any difficulty as to method of computation, ariſe by the other Lines; of 1 or 2 Radiuſſes, by this of 8 you may be preſently reſolved.

Though I muſt needs grant, that the ſingle Line works the Queſtion more exactly, becauſe done to 8 times a greater Radius; yet know that on a Circle of a Foot Diameter, this may be as well done to 8 or 10 Radiuſſes, as to 1 Radius on a 3 foot Rule, which will alwayes approach to 4 Figures if truely made, and ſuch prints you may have ready paſted on a Board at the ſign of the *Sphear* and *Dial* in the *Great Minories London.*

A ſhort Touch of the uſes of thoſe Lines as a little Direction for thoſe that are not acquainted with them.

Firſt for Multiplication.

THe 4 upper Radiuſſes being under or leſs then 1, being cut from the 4 lower Radiuſſes more then 1, and the ſtroke of 1 at the end of the Radiuſſes under 1 being paſted exactly on the 1, at the beginning end of the 4 Radiuſſes, above or more then 1; (or if theſe Prints are too ſmall, you may have them made of any length) then in Multiplication, alwayes the Rule is thus, The extent from 1 in the middle, to ny number over or under 1 being the Multiplier; the ſame extent ſhall reach from the Multiplicand, being laid the ſame way to the Product.

Example, Let ,005 be multiplied by 5, the extent from 1 to 5 ſhall reach, from ,005 to ,025, the Product being but a quarter and leſs then 1.

Again, To multiply 1,79 by 206,50 the extent from 1 to 1,79 being laid the ſame way from 206,50 gives 369,6350, the Rule ſheweth you certainly that 370 is the greateſt Integer though you cannot ſee all the other Fractions, yet this inſures you of the Numbers of places of Integers, and the multiplier and multiplicand by inſpection the right number of Figures as afterwards in uſing the double lines; in the ſame manner for any other Multiplication whatſoever.

Diviſion, the Rule is alwayes thus,

THe extent from the Diviſor to 1 reaches the ſame way from the Dividend to the Quotient; Example, In a Decimal whole Number and Fraction. In 123 *l.* 500 parts, how many Stone 14 *l.* to the Stone? The extent from 14 to 1 being laid the ſame way, from 123 ½ gives 8 ſtone and ,082 parts, then the ſame extent laid the contrary way from ,082 gives 11 *l.* and ½ the remainder in pounds weight under 14 *l.* 1 ſtone.

In the Rule of Three the Method alwayes is

THe extent from the Firſt to the Second, reaches the ſame way from the Third to the Fourth.

Example, If 3 Ounces and ½ of Silk coſt 3 *s.* 4 *d.* what coſt 17 Ounces ¾; the extent from 3 ½ to 3 *s* 4 *d.* in the Money Line, ſhall reach the ſame way from 17 3 fourths, to 16. *s.* and 1 *d.* 3 *ſ.* on the Money Line. Agin,

If the Intereſt of 100 *l* be 6 *l.* in 365 dayes, what ſhall the Intereſt of 20 *l.* be in 40 dayes? The extent from 100 to

40 dayes on the line of time, reaches from 20 *l.* to 2 *s.* 7 *d.* 2 *f.* on the money line: Or you might have counted thus, Juſt againſt 40 dayes on the line of time, on the money line, is 13 *s.* 2 *d.* the Intereſt of 100 *l.* in 40 dayes, then as 100 *l.* to 13 *s.* 2 *d.* ſo is 20 *l.* to 2 *s.* 7 *d.* 2 *f.* as before.

The Back Rule of Three may alwayes be reſolved by the Direct Rule, by a due underſtanding and right ſtating the Queſtion, alſo the Double and Compound Rule at two operations, or by preparing the Numbers, which is onely the Direct Rule twice or thrice repeated, according to the nature of the Queſtion, which cannot be expreſſed now but by many words, too long for this place; therefore for this matter, I refer you to Page 133 of the Triangular Quadrant, or to Mr. *Kerſeys* Arithmetick Page 77.

For the extracting of the Square and Cube-roots, the Rule is alwayes thus by this Line of 8 Radiuſſes,

THe Firſt, Of one or two mean proportions, (Geometrical) between 1 and the number given is alwayes the Square or Cube-root.

Example 1, For the Square-root in two examples,

1. What is the Square-root of 126, 5625 a whole Number and a Fraction? The exact middle between 1 and 126, 5625 will be at 11,25 the Square-root required.

2. What is the Square-root of ,00527 a Fraction? The exact middle between 1 and ,00525 a number leſſer than 1, is ,023 the Square root required.

Secondly , For the Cubick root alwayes thus ,

Example 1 . What is the Cubick root of 1275,63 the firſt of two Geometrical mean proportions between 1 and 1275,63 is at 10,080 , the Cubick root being found by dividing the ſpace on the Line of Numbers (in 8 Radiuſſes) between 1 and 1275,63 into three equal parts exactly , and the firſt beyond 1 is 10,848 the Cube-root required.

2. Again, The Cube-root of ,00053 a number leſſer then 1, is ,0175, for the firſt of two mean proportionals betwen 1 and ,00053 is at ,0175 , found by dividing the diſtance between 1 and ,00053 into three equal parts, counting and uſing the firſt proportional from 1 toward the number propounded.

The Uſe and Application of the Square and Cube-root , is to work proportions in Superficies or Solids, as in pages 107. 108, 109.

109,110, and in page 116, 117, 118, 119 120, 122; and 335, 336, 337 of the Triangular Quadrant, in brief as a taſte thus,

If a fathom of Rope of 6 inches compaſs weigh 6 pound one eighth, what ſhall a fathom of Rope of 12 inches compaſs weigh?

The extent from 6 to 12 being twice repeated from 6,125 ſhall reach to 24,50 the weight of a fathom of Rope of 12 inches about.

On the contrary, The Areas or Square given to find the ſides,

Example is, If the content of a quarry of 10 *s.* be 0,1, and the content of a quarry of 12 *s.* 00,833, divide the ſpace between 0,1 and 00,833 into two equal parts, then that extent laid the ſame way from 6 the length of ſquare 10 *s.* ſhall reach to 5,47 the length of ſquare 12 *s.* and from 4 the breadth of ſquares 10 *s.* to 4.38 the breadth of ſquare 12 *s.*

Example again for Solids , If an Iron Bullet 6 Inches Diameter weigh 30 pound what ſhall a Bullet of 8 Inches Diameter weigh? The extent from 6 to 8 ſhall reach at three repeatings from 30 to 71 one third , on the contrary one third part of the extent on the numbers from 71 one third to 30, ſhall reach from 8 to 6 the Diameters of the Shot ; and ſo for any other the like.

But note , That the trebble and ſingle Line is quicker and eaſier, and ſaves the trouble of dividing into two or three parts ; for the extent from 6 to 8 on the ſingle Line , reaches from 30 to 71 one third on the trebble line , and the contrary.

Laſtly, For ſimple Intereſt, by the line of Time and Money.

Count the Time on the line of Time , from a day to 100 year , and juſt againſt it on the Money line, is the Intereſt of 100 *l.* for that time, and for any other ſum thus , The extent from 100 on the Numbers to the time counted on the line of time, ſhall reach the ſame way from the ſum of money propounded to the intereſt thereof the ſame time.

Example, The extent from 100 counted on the Numbers to 6 *l.* counted on the money line, ſhall reach the ſame way from 30 *l.* counted on the Numbers to 1 *l.* 16 *s.* the intereſt due for 30 *l* in 12 months time:

But for this matter there is enough in the following diſcourſe and the work is beſt done by the larger lines; but to ſee the true Number of Places, and ſee the increaſe, and the decreaſe of Geometrical porportion, the line of 8 or more Radiuſſes will prove an excellent help to Learners, for whoſe ſake it is here inſerted.

THE DESCRIPTION OF THE Joynt Rule.

The lines on this Rule, or the lines made in part or in the whole may be varied according to any man's particular uſe or inclination in ſome part thereof: yet a deſcription muſt be of ſome thing, that which my preſent thoughts calls moſt convenient is as followeth.

1. First, next to *inches* in 8 parts for *one foot* is annexed the *foot meaſure*, for ready *Reduction* and alſo for *Menſuration*.

2. Next the other part is ſet the *price* of *one foot* of *Brick-work* at any value per *rod* in the nature of a Scale.

3. Then on the ſame flat-ſide, as lines on a *Sector* drawn from a *center*, is a ſcale to 30, and every integer parted into 12 parts to repreſent *inches*. or every 5th *minute*, every integer being a *model*.

4. Next to this is another ſcale of the ſame length divided into 40 integers or feet.

5. Next to, or as neer as may be to the 30 ſcales on the head leg, is added a line of *Natural Sines*, but Figured as the line of *Chords* to 180 degrees, to meaſure any Angle readily; to which is added a line of *Tangents* in pricks to 63,30, for ſeveral uſes.

6. On the inner line of the 30 ſcales are ſet points to divide a Circle into 20 or more or any number of parts.

7. On the edge of the Rule is ſet the line of *Numbers*, to which is added the decimal Fractions of a pound *ſterling*, the inches in a foot to 12 foot, and ſeveral points for ſeveral uſes more than formerly have been thought on.

8. On the inſide is ſet the ordinary

lines of *Timber* and *Board* meaſure.

9. On the other flat ſide is ſet the lines proper to a *Quarant*,

As the *degrees*, the line of *ſhadows*, the *months* and dayes of the *year*, the *Suns declination*, *riſing*, and *ſetting*; his *hour*, and *Azimuth*, *Amplitude ortive* and *occaſive*:

And a perpetual *Almanack*: All theſe on one leg as by their names appear.

10 On the other is ſet a *line of lines*, or *equal parts*; a line of *Natural ſines* to find the *hour* and *Azimuth* generally, and a line of *ſines* for a perpetual *Latitude*, as the ſeveral names manifeſt.

11. On the ſpare place is ſet the ſcales for *Diameter*, *Circumference* and *Square equal* and *inſcribed*; or a *Tide-table* of 24 hours, and the *Moons* age, or any thing elſe thought convenient, the uſes of all which are here briefly and plainly delivered.

Uſe

Uſe I.

Of Inches and Foot meaſure.

The uſe of Inches, and Foot meaſure as to taking of Dimenſions, is here needleſs: Reduction by thoſe Lines from 10 *to* 12 *and the contrary, is obvious to every conſiderate obſerver.*

FOR as 6 is half 12, ſo 50 is half 100, and conſequently 3 foot 7 inches and 7 eights to be reduced to decimals, it will be, 3, 656: for juſt againſt 7 inches and $\frac{7}{8}$ is on the Line of Foot meaſure 656, the 65 is expreſſed and the laſt 6 eſtimated.

And on the contrary, if 3 foot and 46 parts were to be reduced to inches and eight parts, look for 46 on the line of foot meaſure, and juſt againſt it on the Inches, is 5 inches and $\frac{4}{8}$ the anſwer.

The ſame holds in money or any thing, where the integer is parted into 12 parts, is 12 pence in a ſhilling, or 12 dozen in a groce, or the like.

Uſe II.

The price of 1 of any Commodity given, to find the price of 100.

FIrſt, for any rate under 100 ſhillings or 5 *l.* per 100, count thus,

Count the ſhillings per 100 on foot meaſure, and right againſt it on the inches is the price of one in pence and farthings

Example, *At* 48 *ſhillings* per 100 *what coſt* 1?

Seek for 48 ſhillings on foot meaſure and right againſt it on inches is 5 *d.* 3 farthings, the Anſwer required.

On the contrary at 7 *d* for one counted at 7 inches on foot meaſure is 58 *s.* 3 *d* the price of 100.

In this counting every inch is 1 penny, but if the ſum be between 5 *l.* and 10 *l.*

per 100, then call 10 on the foot meaſure 1 *l.* and every half inch 1 penny.

Example, At 7 *l.* 8 *s.* per 100 counted at 74 on Foot meaſure, on the inches is 17 *d.* 3 farthings by doubling the inches or counting every half inch for a penny.

Again, on the contrary, at 18 *d.* for 1 being counted at 9 inches, right againſt it on foot meaſure, is 7 *l.* 10 *s.* per 100.

But if the ſum be above 10 *l.* *per* 100 or ſquare, then this is the beſt way Double the 10 *s.* of pounds, and the ſum call ſhillings double alſo the units of pounds, and if the ſum be above 10, add 1 *s.* more, and the remainder call 10 *s.* of ſhillings, to which add the remaining ſhillings and pence if any be, and ſeek it on the line of foot meaſure, and juſt againſt it on the inches is the remaining pence, and 8 parts of a penny required: Example, At 72 *l.* 15 *s.* 10 *d.* *per* ſquare or 100, what comes 1 foot (or one) to in money?

Firſt double 7 the tens of pounds and it is 14 *s.* then the two pounds counted s 10 *s*. of ſhillings is 40 *s*. and the 15 *s*. ver makes 55 *s*. and 10 *d*. which 55 *s*. nd 10 *d* ſought on foot meaſure on the nches is 6 *d* 2 farthings, and 3 quarters f a farthing, in all 14 *s*. 6 *d*. 2 *f*. ¾ the xact price of 1 foot (or 1) at 72 *l*. 15 *s*. 10 *d*. *per* ſquare or 100.

Again On the contrary, the price of 1, or 1 foot being given, to find the price f 100 or a ſquare.

Halve the ſhillings, and reduce the pence nd farthings to decimals by the foot meaſure, and then halve that alſo, and the um is the anſwer.

Example, at *9s*. 9 *d*. 2 *f*. ½ per foot what omes a ſquare to ·

The half of 9 is 4 for 40 *l*. and car-y 1. Then 9 *d*. 2 *f*. ½ farthing reduced y foot meaſure is neer 80 to which adding 100 for the 1 carryed, makes 180, hole half is 90, for 9 *l*. in all 49 *l*. per quare, the exact Anſwer required.

Or count thus, For every ſhilling *per* oot count ſo many hundreds, then the ence and farthings being reduced to decimals,

cimals by foot meaſure, add to the hu dreds, and the half of that ſum is th price of 1 ſquare; or 100 bein halved after the way of bringing of ſhi lings to pounds in this manner,

Example, at 15 *s.* 6 *d.* 2 *f* per fo what comes 100 to?

Firſt, for 15 *s.* ſet down ——	15
Then 6*d* 2 *f.* reduced by foot meaſure is ——————	5
The ſum added is ————————	155
The half is 77 *l.* 14 *s.* the exact price of a ſquare ——	77 *l.* 14

Uſe III.

To find the exact price of 1 *foot of Bri work at any price per Rod* 272 *foot being a Rod, or the contrary, havi the true price of one foot, to find t price of a Rod.*

ON the other half of the Rule, vi from 12 inches to 24 next the hes, is ſet a ſcale of *equal parts* to 10 nd [illegible], neer 24 inches, whoſe uſe i thus, At 4 *d* *per* foot of brick-work, what omes 272 foot to?

Seek 4 pence which is at the 4 on his ſcale, and juſt againſt it on the inches is 4 *l.* 11 *s.* the price of a Rod, or 272 *f.* $\frac{1}{4}$. Note in this account every inch s 1 pound, and every 8th part is 30 *d* or 2 *s.* 6 *d.*

Again, at 5 *l.* 2 *s.* 6 *d.* *per* Rod, what comes 1 foot to? Juſt againſt 5 inches nd $\frac{1}{8}$ counted in the inches, on the line annexed is 4 *d.* 2 *f.* the exact price of a foot of Brick-work. When the price is above 12 pound per Rod, then call 6 *l.* 12 *l.* and 5 *d.* 1 *f.* againſt it 10 *d.* 2 *f.* and you ſhall be reſolved to 24 *l.* *per* Rod

Uſe IV.

By the Line of Numbers and line of Pen annexed to work the former or any oth queſtion in the Rule of Three or Pra ctice.

I.

At any price per 100 *What coſt* 1?

1. FOR any price *per* 100 under 2 count thus, ſeek the price in th line of pence, and that is the exact an ſwer to the Queſtion.

Example, At 3 *d.* per 100 the price o 1 is $\frac{3}{100}$ part of a penny into a hundre parts.

Again, at 12 *d.* per hundred found a 12 *d* on the line of pence, the Anſwer i 12,100 parts of one penny, being nee half a farthing.

2. For any price *per* hundred betwee 2 *s.* and 10 *l.* *per* hundred, count thu

Count 1 at the beginning for 2 *s.* in the middle for 1 *l.* and 10 at the en

ſo

r 10 *l.* Example, at 16 *s.* 8 *d.* per hun- red what coſt 1?

Note, if 1 at the beginning be 2 *s.* and in the middle 20 *s.* then 16 *s.* is at 8 eer the middle 1 and the 8 *d.* count 4 welve parts forward on the line of Numbers: then juſt againſt it, on the ine of pence is 2 *d.* the true anſwer required.

Again, at 2 *l.* 10 *s.* per 100 is 6 *d.* or 1.

3. From 10 *l.* to 1000 *l.* per 100, ount thus, count the tens of pounds in he firſt part of the line of Numbers, and double it for ſo many ſhillings, and that is the anſwer in ſhillings.

Then count the unites of pounds and ſhillings and pence more on the line of Numbers, and right againſt it on the line of pence is the pence and farthings more to be added to the ſhillings firſt found out. Example,

At 72 l. 10 s. 6 d. *per ſquare, what comes* 1 *foot to?*

Firſt, 7 the tens of pounds doubled

is 14 *s.* the anſwer in ſhillings, then 25,25 the decimal of 2 *l.* 10 *s.* 6 *d* over ſought on the line of Numbers; juſt againſt it on the line of pence is 6 *d.* 0. *f.* 1 *quar* in all 14 *s.* 6 *d.* 0 *f.* $\frac{1}{4}$ the exact anſwer and price of 1 foot at 72 *l.* 10 *s.* 6 *d.* per ſquare.

Again, at 325 l. *per ſquare, what coſt* 1 *foot?*

Firſt, the double of 32 the tens of pounds is 64 *s.* then juſt againſt 5 the pound over ſought on Numbers is juſt 12 *d.* in all 65 *s.* per foot at 325 *l.* per 100.

1. *On the contrary, the price of one foot being given to find the price of a ſquare or* 100 *foot.*

Example, at 18 *s.* 7 *d.* 2 *f. per foot What comes a ſquare to?*

Firſt, ſeek for 18 *s.* which is at 9 on the firſt part of the line of Numbers, and that 9 then ſtands for 90 *l.*

Then the odd 7 *d.* 2 *f.* ſought out on the line of pence on the numbers, right againſt it is 31,25 or 3 *l.* 2 *s.* 6 *d.* in all

93

93 *l.* 2 *s.* 6 *d.* per ſquare, the exact anſwer.

Again, at 7 l. 5 s. *per foot, what coſt* 100?

In 7 *l.* is 140 *s.* to which adding 2 Cyphers makes 14000 *s.* or 700 *l.* then adding two Cyphers to 5 *s.* make 500 *s.* or 25 *l.* in all 725 *l.* the price of one ſquare at 7 *l.* 5 *s.* per foot.

2. For all Rates under 2 *s.* per foot, ſeek the price in the Line of Pence, and in the numbers juſt over is the price of the ſquare required.

Example, At 18 *d.* per foot, the price of the ſquare or 100 foot is 7 *l.* 10 *s.*

Note, Theſe pence and farthings read on the line of Numbers, are only the Decimals of thoſe pence and farthings, 1 *l.* being the Integer, which you muſt conceive to be a Radius or Revolution further beyond 10 at the end; thus the Decimal of 4 *d.* 2 *f.* is 0,01875.

The Decimal of 3 half pence is 0,00625.

The Decimal of 3 *s* 6 *d.* is 0,175.

Uſe V.

The uſe of the Line of Numbers in ſeveral Rules, as Multiplication, Diviſion, Reduction, Rules of Three, *&c.*

I. *Multiplication by the Line of Numbers.*

IT is ſuppoſed, that the way of reading the Line is well known already, not, conſult the Carpenters Rule, or the firſt Chapter of the Triangular Quadrant: in brief thus, if 1 at the begining be called 1 Integer, then 1 in the middle is 10 Integers, and 10 at the end is 100 Integers, and the cuts betwen the Decimal, Centeſſimal and Mileſſimal parts between, as the room will allow.

In Multiplication 1 is alwayes the firſt term, the Multiplicator (or Multiplicand the ſecond, it matters not which, but moſt properly the Multiplicator) the ſecond, the Multiplicand the third, and the Product the fourth.

The Operation alwayes thus,

The extent from 1 to the Multiplicator being laid the ſame way from the multiplicand ſhall reach to the Product.

Example, Let 125 be multiplied by 75, the extent from 1 to 75 being laid the ſame way from 125 will reach to 9375.

Thus for any other ſum, but note that to 4 Figures is as many as you can ſee on moſt Lines of Numbers.

But to know the certain Number of Figures, there is for the moſt part as many in the Product as in the Multiplicator and Multiplicand both added; and ſometimes 1 leſs, which is, when the firſt Figures of the Product are greater, then the two firſt Figures of the Multiplicator, or Multiplicand.

2. Note alſo, when the Product conſiſts

ſiſts of more than 4 figures, to juſtifie the laſt, uſe this help; ſet down the Numbers as if you would multiply it by the Pen, then with Inke or Chaulk begin to multiply the Numbers as in the Example; ſuppoſe I were to multiply 168 by 249.

Then by the firſt note I know I muſt have 5 or 6 figures in the Product.

Then to juſtifie the two or three laſt figures, the Numbers being ſet down, begin to multiply, ſaying, 8 times 9 is 72, ſet down 2 and carry 7; again, 8 times 4 is 32 and 7 is 39.

```
  249
  168
 ----
 1992
1494
  9
 ----
  832
```

Again, 6 times 9 is 54, ſet down 4, and carry 5, 6 times 4 is 24 and 5 is 29.

Laſtly, if you will, once 9 is 9, then thoſe three firſt Figures added are 832.

Then the extent from 1 to 168 being

laid the ſame way from 249, gives 418 plainly, and the 32 is juſtified by the ſecond note and number of figures, *viz.* 5 and not 6, is becauſe in 41832 the Product 41 is more than 24, or 16, the two firſt Figures of the multiplicator, or multiplicand; by well heeding, and frequent practice this will be ready to any ordinary capacity.

3. In Multiplication of whole Numbers, and Fractions, or Fractions only; it is all one by the line of Numbers as whole Numbers, and for the Fraction of a Pound *Sterling*, or the Pence in a Shilling, or the Inches in a Foot, they are expreſt in Pricks as much as needs, as you will fully ſee in Rules of Practice.

4. *Note*, That as Multiplication of Integers do increaſe, ſo Multiplication of Fractions do decreaſe and become leſs then 1, for once 1 is but one; therefore leſs than 1 multiplyed by leſs then 1 muſt needs be leſs then 1.

Uſe VI.

Of Diviſion.

1. IN *Diviſion* the Diviſor is alwayes the firſt term, and 1 the ſecond; the Dividend the third, the Quotient the fourth: *Example*, let 523 be divided by 17.

2. The extent from 17 to 1 ſhall reach the ſame way from 523 to 30, and 764,of 1000 the true Quotient and Decimal Fraction required, which Decimal Fraction if you would reduce to the Vulgar Fraction found by the Pen, then lay the ſame extent of the Compaſſes the contrary way, from the Decimal Fraction laſt found and it gives 13,17 the remainder found by the Pen.

For firſt the extent from 17 to 1, laid the ſame way from 523, gives 30 for Integers, in the Quotient, and 764 more, as a Decimal Fraction.

Laſtly, the ſame extent laid the contrary way from 764 the decimal Fraction gives 13 the Numerator of 17 the vulgar Fraction found by the Pen.

3. If you divide Mixt Numbers, *viz.* Integers and Fractions, by Integers and Fractions, or divide leſſer Numbers by greater, is as eaſie by the Line as Integers only; eſpecially, uſing the line of 8 Radiuſſes, which doth inſure you of the number of places.

Example,

Let 17 *s.* 6 *d.* be divided by 7,5, as among 7 Men and a Boy, counting the Boy half pay.

The extent from 7,5 to 1, laid the ſame way from 17,5, ſhall reach to 2,333 each mans ſhare, the half of 2,333, *viz*, 1, 1666 being the Boyes ſhare; for if 2,333 be multiplyed by 7, the Product is 16,333 to which adding 1,166 the ſum total is near 17,5.

4. Or by the Line of Pence and Farthings thus, with the Line of Numbers

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4. Or by the Line of Pence and Farthings thus, with the Line of Numbers

The extent from 7,5 on the Numbers to 1, ſhall reach from 17 *s.* 6 *d* to 2 *s.* 4 *d.* each man's ſhare, the half of which *viz.* 1 *s.* 2 *d.* is the Boyes ſhare.

Example, Of a leſſer Number to be divided by a greater. If 2 *l.* 6 *s.* 10 *d.* is to be divided among 52 men, and to allow one eighth part for the Clerk, how much is each mans ſhare?

The extent from 52 one 8th. to 1 counted on the Line of Numbers, ſhall reach from 2 *l.* 6 *s.* 10 *d* counted on the Numbers, to 10 *d.* 3 farthings, the anſwer on the Line of Pence.

5. The number of Figures in any Quotient, is as many as the figures of the Dividend exceed the figures of the Diviſor, and one more, when the firſt figure of the Dividend is greater than the firſt of the Diviſor.

Uſe VII.

Of Reduction.

THe general Rule is, As one Denominator to the other Denominator; ſo is one Numerator to the other: *Example*, To reduce Ounces to a Decimal Fraction, as 16 to 1000 ſo is 12 Ounces to 750, the Decimal Fraction required.

But Inches and Deicmalls, Pounds and Decimals are reduced by the Lines of Inches and Foot Meaſure, and by the Line of Numbers, and Line of Pence by inſpection only; for if you would have a Decimal for 15 *s.* 6 *d.* read thus, 15 *s.* is 75, and juſt againſt 6 *d.* in the Line of Pence on the Line of Numbers is 025, in all ,75,25. And on the contrary, 7 double is 14 *s.* and 5 the next figure, the half of 1 in the ſixth, place is 1 *s.* and 25 a quarter of 2 *s. viz.* 0,1 is 6 *d.* the ready Reduction required.

Uſe VIII.

To find the Square Root of a Number, being a Geometrical mean Proportion between it and 1.

1. FIrſt conſider, whether the Number conſiſts of even or odd Places, *viz.* of 2, 4, 6, or 8 places, or 1, 3, 5, or 7 Places.

2. It is convenient to ſet the number down in Ink or Chalk, and Point it, as in the Example, to find the true number of Places in the Root; for look how many Points, ſo many places in the Root.

3. When the Number of places are even, then the Unite or 1 is counted at the right end; when 10 is uſually ſet, and the Number and Root read toward the

the left hand. But when the number of Figures or Places be odd, then the Unite is to be accounted at 1 on the left end, and the Root and Number counted forward toward the right hand.

Example, To find the Square Root of theſe two Numbers, *viz.* 8464 and 17424, firſt Point them as in the Example, at the firſt, third and fifth place, being read from the Unite or right hand, then firſt for 8464 conſiſting of even places, the exact middle, between 10 at the right end, and 8464 counted leftward, will be found at 92 the Square Root, and becauſe the Number is Pointed with two Points, it conſiſts of two places of Integers.

Again, for 17424 a number of odd places and three Points, the left hand or middle 1 is the Unite, and the Root and Number counted toward 10 forward.

The exact middle between 1 and 17424, read as neer as you can, will be at 132, of 3 places, becauſe the Number had 3 Points.

Theſe Rules are as to the Integers, the Fractions are Decimals.

Uſe IX.

Of the Cube Root.

The Cube Root is the firſt of two mean Proportions Geometrical *between* 1 *and the Number given*; found thus,

1. First ſet the Number down and point it at the firſt, the fourth, the ſeventh, and tenth Place counting from the Unite or Right hand toward the Left. And note, how many Points, ſo many Figures in the Root.

2. When the Point falls on the laſt Figure, then the Middle 1, or Left hand 1 is the Unite, and the Root and Cube counted forward toward 10.

3. When the point falls on the laſt but one, then the Unite may be at either end, as at 1 at the beginning, or left end, or at 10 at the right end; but the Cube will be in a Radius and more beyond 1 either forward or backwards, ſo that to work it, the numbers muſt be here repeated as is uſual.

4. But when the Point falls on the laſt Figure but two, then 10 at the right end is the Unite, and the Root Square and Cube is found all backward, in the ſame Radius, *viz.* between 10 and the middle 1.

Example, in theſe three Numbers,

1. A third part between 1 and 5832 is 18 the Cube Root required, found between 1 and 10.

2. A third part between 1 and 32768 is at 32 the Cube Root found in the ſame Radius beyond 1 forward, or behind 10 backward; but the number is counted in a Radius beyond 1 forward, or behind 10 backward.

3. A third part between 10 and 474522.

474522 is at 78, between 10 and the middle 1 backwards.

Uſe X.

Of the Rule of Three.

IN Queſtions of the Rule of Three, the chief difficulty is in ſtating and underſtanding the Queſtion, to Methodiſe it well; as thus, to find out the firſt, ſecond, and third Terms; which may be done thus,

1. In all Queſtions of the Rule of Three, three Terms are given, and a 4th, demanded: of the three given Terms, two are of ſuppoſition and one of demand or enquiry, being alwayes the third Term.

2. Of the three Terms, two are of one Denomination, and one of another; (or are made ſo by Reduction)

3. Alwayes the Term of Demand is

the third Term, (in Direct Proportion) to one of the other Terms of Suppoſition, *viz*, that of the ſame Denomination with the third Term, is the firſt Term; then the other left muſt needs be the ſecond; the anſwer to the Demand the fourth Term.

4. This being premiſed, the General Rule by the Line of Numbers is,

The extent of a pair of Compaſſes from the firſt Term to the ſecond, counted on a Line of Numbers, ſhall reach the ſame way, on the ſame Line of Numbers, from the third Term to the fourth.

1. *Example*, If one foot of Timber coſt 10 *d.* what coſt 35 foot? The extent from 1 at the beginning to 10 counted at the middle 1, ſhall reach the ſame way from 35 counted in the firſt part, to 350 Pence counted in the ſecond part.

2. Again, to bring 350 pence to Shillings,

The extent from 12 to 1, ſhall reach the

the ſame way from 350 to 29 *s.* 2 *d.*

3. Or uſing the Line of Pence, The extent from middle 1 on the Line of Numbers to 10 *d.* in the Line of Pence ſhall reach on the Numbers from 35 to 1 *l.* 9 *s.* 2 *d.*

The Reverſe and Double Rule, or Compound Rule of Three, by the Line of Numbers, are of no account, for the Reverſe Rule, by altering the ſtate of the Queſtion, may be reduced to the Direct Rule; and two ſingle Rules wrought ſooner then the Numbers ſhall be prepared for the Compound Rule; but of them may you ſee more in ſheet K of the Triangular Quadrant.

Uſe XI.

Of the Rule of Practice.

THis Rule is only the Rule of Three variouſly applied, wherein the naming the Queſtion requires more words than the working and the great care in ſtateing, whereby the reaſon of the Rule may appear, as is well ſet forth in Page 285 of Mr. *Kerſeys* Arithmetick.

But theſe Rules of Practice by the Line of Numbers, ſo far as the Line will expreſs, are wrought without ſuch burthen to the Memory, any more than needs to underſtand the Queſtion; as in theſe ſeveral Examples,

1. If one Yard or Pound coſt 3 *d.* what coſt 36 Yards? Anſwer, 108 Pence.

For the extent from 1 to 3, ſhall reach from 36 to 108 pence.

Or the extent from 1 on the Line of

Numbers to 3 *d*. on the Line of Pence, being laid the ſame way from 35, ſhall reach to 45 or 9 *s*. 100 being 20 *s*. or one Pound, as is made plain in Uſe 4th.

2. If 3 and 3 quarters of any thing ſhall coſt 7 *d*. 3 far. what coſt 39 ½ of the ſame Commodity? Anſwer, 6 *s*. 9 *d*. 3 *f*. for the extent from 3,75 to 7 *d*. 3 *f*. on the Line of Pence, being laid the ſame way from 39,50 gives 3,410.

Note, The 3 reduced is 6 *s*. then 410 counted on the line of Numbers, juſt againſt it on the Line of Pence is, 9 *d*. 3 farthings.

3. If 112 Pound coſt 28 *s*. what coſt 1 *l*? The extent from 112 to 1 *l*. 8 *s*. counted on the Line of Numbers, laid the ſame way from 1 on Numbers, gives 3 *d*. on the Line of Pence.

4. If 1 *l*. coſt 5 *d*. 2 *f*. what coſt 112? The extent from 1 to 5 *d*, 2 *f*. being laid from 112, gives 2 *l*. 11 *s*. 4 *d*.

5. If 112 *l*. coſt 2 *l*. 10 *s*. 6 *d*. what coſt 21 *C*. 3 *q*. 12 *l*. Anſwer, is 55 *l*. 4 *s*. 0 *d* for the extent from 112 to 2,525

being

eing laid the ſame way from 2448 the ounds, in 21 *C.* 3 *q.* 12 *l.* ſhall reach to 5 *l.* 4 *s.*

Or the extent from 1 to 2,525 being id the ſame way from 21,86 ſhall reach o 55 *l.* 4 *s.* as before.

6. If 5 Ounces ½ coſt 20 *d.* 3 *f.* what oſt 1 *C.* 1 *q.* and 15 *l*? The Anſwer is 9 *l.* 2 *s.* 10 *d.* wrought at twice, thus,

1. The extent from 5 ½ to 20 *d.* *f.* laid the ſame way from 16 the unces in 1 *l* gives 2518, or 5 *s.* 0 *d.* 2 *f.* ear.

2. The extent from 1 to 2518 being laid from 155 the pounds is 1 *C.* *q.* and 15 *l*, to 39 *l.* 2 *s.* 10 *d.*

Uſe

Uſe XII.

For meaſuring Board or any Superficial Meaſure.

1. AS 12 to the Inches broad, ſo is the length in feet to the Content in feet; or as 1 to the breadth, ſo is the length to the Content in Foot Meaſure.

Example, At 8 Inches broad and 20 Foot long, the Anſwer is 13 Foot 4 Inches.

2. As the breadth in Inches to 12, ſo is 12 to the Inches that make 1 Foot.

Or as the breadth to 1, ſo is 1 to the length of a Foot in Foot meaſure.

3. As 1 Inch to the breadth in Inches, ſo is the length in Inches, to the Content in Inches.

Or as 144 to the breadth in Inches, o is the length in Inches, to the Con- ent in Feet.

Or by Foot Meaſure, as 1 Foot to the ength in Feet and Parts; ſo is the breadth n Feet and Parts, to the Content in eet and Parts.

4. To meaſure any Triangle Figure.

As 2. To the whole Perpendicular, o is the whole Baſe of any Triangle, o the Superficial Content in like mea- ure.

5. All Irregular Figures are beſt nd eaſieſt meaſured when reduced to Triangles or Trapeſiaes, and then caſt p as Triangles or Parralelograms, as uſt before.

6. To find a Square equal to any ong Square, divide the ſpace on the ine of Numbers, between the length nd breadth, into two equal parts, and he middle ſhall be the ſide of the ſquare qual to that long ſquare.

Example, Let a long Square be 4 oot broad and 9 foot long, the mid-
dle

middle between 4 and 9 on the Line of Numbers will ſtay at 6, and 6 is the ſide of a Square equal in Area to a long Square of 9 foot one way and 4 foot the other; for 4 times 9 is 36, and 6 times 6 is 36 alſo.

Uſe XIII.

To Meaſure Solid Meaſure.

1. THE middle between the breadth and thickneſs, is the ſide of a Square equal.

2. The ſide of the ſquare equal being known, to find how much in length makes one foot of Timber.

The extent from the ſide of the Square to 1. being turned twice the ſame way from 1, ſhall reach to the length required to make a foot of Timber in foot Meaſure.

Or the extent from the inches Square

to 12, being turn'd twice the ſame way from 12, ſhall reach to the inches in length to make a foot of Timber.

Example, At 50 of a foot in 100 or 6 inches Square.

The extent from 5 to 1, being twice repeated from 1 reaches to 4, for 4 foot.

Or the extent from 6 to 12 being twice repeated from 12, ſhall reach to 48 inches the length in inches required.

But if the Timber or Stone be not Square or Squared, then ſay,

As 12 to the breadth in inches, ſo is the depth in inches to a fourth:

Again, as the fourth is to 12, ſo is 12 to the length required in inches, to make one foot of Timber.

Example, At 9 and 4 inches broad and thick.

The extent from 12 to 9 ſhall reach the ſame way from 4 to 3, a fourth.

Again, the extent from 3 the fourth to 12, ſhall reach from 12 to 48, the inches in length to make 1 foot.

The ſame work ſerves for foot Meaſure uſing 1 inſtead of 12.

3. *The ſide of the Square, or the breadth and depth given, to find how much is in 1 foot long; 3 wayes.*

1. As 12, or 1, to the ſide of the ſquare in inches, or foot meaſure, ſo is the ſide of the ſquare, to the quantity in a foot long required.

2. The extent from 1 to 9 inches the breadth, ſhall reach from 4 the depth to 36 the long inches in a foot long.

3. The extent from 12 to 4 the depth, ſhall reach from 9 the breadth to 3 twelfth parts of a foot of Timber, the quantity in a foot long at thoſe dimenſions of breadth and depth.

4. *The ſide of the ſquare and length in feet given, to find the Content in feet.*

The extent from 12 to the inches ſquare, being twice repeated from the length in feet gives the content in feet.

Or the extent from 1 to the ſide of the Square in foot meaſure, being twice repeated from the length in foot meaſure, gives the content in foot meaſure.

Or if the Timber be not Square or ſquared, ſay,

As 12 or 1 to the depth, ſo is the breadth to a fourth.

Again, As 12, or 1 to the fourth, ſo is the length to the content required.

Example, at 15 inches deep, and 18 inches broad, and 13 foot long.

The extent from 12 to 15, ſhall reach from 18 to 22,50: Again,

The extent from 12 to 22,50, ſhall reach from 13 to 24 foot 38 parts, or 4 inches and a half the Content.

5. In meaſuring of Taper Timber, to the content found by the middle Square or girt, add a piece, the ſide of whoſe ſquare ſhall be half the difference of the ſquares at each end, and the length one third part of the whole length, and thoſe two ſums ſhall be the true content.

Uſe XIV.

For Round Timber.

1. AT any Diameter given, to find how long makes one Foot: The extent from the Diameter to 46 90 being twice repeated from 1 gives the length to make a foot of Timber in inches.

Or the extent from the Dtameter to 13, 54 being twice the ſame way from 12, ſhall reach to the inches long to make a foot.

2. At any Diameter given, to find how much is in a foot long.

The extent from 13.54, to the Diameter in inches, being twice repeated from 12, gives the quantity in 1 foot long, in inches.

But for great Timber ſay, ſo is 1 to the Content of 1 foot long in feet and parts.

3. The Diameter given in inches, and the length in feet, to find the whole Content in feet.

The extent from 13,54 to the Diameter being twice repeated from the length in feet, gives the ſolid Content in feet.

If you work by the Circumference there the Center uſed is 134,5, the inches about, when one foot long makes a foot.

Uſe XV.

Of a Circle.

For the ready Meaſuring of a Circle and its parts uſe theſe Centers by the Line of Numbers

Diameter	10	0
Circumference	31	42
Square equal	8	862
Square within	7	071
Area of a Circle	78	54
Area of a Semicircle	39	27
Area of a Quadrant	19	635

THE uſe of theſe Points is thus, Having any one of theſe given, to find the reſt.

The extent from 10 the fixed Diameter, to the given Diameter, laid the ſame way for the fixed Circumference reaches to the Circumference required.

But for Area's of Circles, half Circles, or quarters, you muſt turn the Compaſſes twice.

Example, The extent from 10 to 30, being repeated twice the ſame way from 78.54 gives 707, the Area of a Circle whoſe Diameter is 30.

If the Area's be given, and a ſide quired, the half diſtance between the Area's, laid from the Centers, for the ſide given, gives the reciprocal inquired ſide.

Example, The half diſtance between 707, and 78,54, laid from 30 gives 10, or from 10 gives 30, for the Diameters required.

For an Oval or ſuch like Figure the middle between the Tranſvers (or longeſt) and Conjugate (or ſhorteſt) Diameters, meaſured on the line of Numbers, gives the Diameter of a Circle equal to the Oval.

For a Sphear or Globe.

THE Diameter and Circumference is found as that of a Circle is; the ſuperficial and ſolid content find thus,

The extent from 1 to the Diameter being twice repeated from 31,42 ſhall give the Superficies round about.

And the extent from 1 to the Diameter being thrice repeated from 523,6 ſhall reach to the Solid Content required.

Example, At 30 inches Diameter,

As 10 to 30, ſo is 31,42 twice to 2827--8.

As 10 to 30 ſo is 523,6 thrice to 14145 the Solid Content of a Globe of 30 Inches Diameter, in like Solid Cube Inches.

To meaſure a Segment of a Globe.

As 1. to the Segments Diameter, ſo

is half the Altitude twice to the Solid Content neer.

In meaſuring Taper Timber find the difference of the Squares at each end, then to the Content found by the middle Square or Diameter add another piece, whoſe ſide of the Square is half the difference of the two Squares, and whoſe length is 1 third part of the whole length.

A Solid Oval reduce to a Globe by a mean between the Tranſverſe and Conjugate Diameters, as before in the Superficial Oval.

Uſe XVI.

Of Gauging.

GAuging is the meaſuring of any Solid Body, or Veſſel, and giving the quantity in Gallons, Wine or Ale Meaſure, or in Beer, or Ale Barrels, or any other greater or leſſer quantities, and for ſeveral forms of Cask, are ſeveral meet Rules and Proportions for operation; ſome of the moſt uſeful are here inſerted.

A wine gallon	contains	231	Cube inches which ſerve as Diviſors.
An ale gallon	288 or	282	
An ale barrel	9216 or	9024	
A beer barrel	10368 or	10152	
A corn gallon		272¼	
A corn buſhel		2178	

Of Gauge Points for theſe Meaſures both for round, and alſo for ſquare Veſſels.

Names of the Gallon Veſſels		G. P. Round	G. P. Square.
Gallon	Wine	17—19	15—199
	Corn	18—52	16—500
	Ale at 282	18—95	16—792
	Ale at 288	19—15	16—[illegible]72
Corn	Buſhel 272 1,4 h	52—66	4[illegible]—568
Ale	Barrel at 282	107—20	95—036
Beer	Barrel at 282	113—69	100—758

The Gauge Points for round Veſſels, are the Diameters of thoſe Circles whoſe Area's are equal to the Cube Inches in thoſe meaſures.

The Gauge Points for Square Veſſels are Square Roots of thoſe Numbers of Cube Inches in each Veſſel, or Meaſure, as Gallon, Buſhel or Barrel.

The uſe of thoſe Diviſors and Gauge Points are briefly thus,

Having multiplied the length in inches, by the breadth in inches, and then that Product by the inches deep, this ſecond Product divided by the Cube Inches contained in any Veſſel, gives the anſwer to your queſtion required.

Or more briefly by the Gauge Points, Example, *both for ſquare and round Veſſels; and firſt Square,*

The extent from the Gauge Point to the length, laid the ſame way from the breadth, gives a fourth.

The extent from the Gauge Point to the 4 laid the ſame way from the depth, gives the Content in Gallons, Buſhels, or Barrels, as the Gauge Point was.

But if the Veſſel be ſquare or ſquared, ſay; As the Gauge Point to the ſide of the Square, ſo is the depth twice to the Content in like meaſure.

Example, The Equated Diameter of any round Veſſel being 28,1 and the length 40 inches, the Content will be 107,54 Wine Gallons.

The ſame manner of operation ſerves to uſe any other Gauge Point, either for ſquare or round Veſſels; how to come by the Equated Diameters, conſult other Books of Gauging, or Experience, as in the *Triangular Quadrant* is more at large explained; in brief, That as 10 to 7, or 6,9, 6,8, 6,7, 6,6, or 6,5. as Experience guides; ſo is the difference of Diameters to a fourth to be added to the leſſer Diameter.

Uſe

Uſe XVII.

To meaſure Brick-work, and reduce it to Brick and half at one Operation.

FOR this purpoſe there are Points ſet for half a Brick, 1 Brick, 1$\frac{1}{2}$, 2, and 2 and $\frac{1}{2}$, and 3 Bricks at 30, 15, 10, 7,5, 6, and 5; and uſed thus.

The extent from 30 to the breadth, ſhall reach from the length of any Wall half a Brick thick, to the Content of at a Brick and a half thick.

Example, at 12 foot high and 20 foot broad at 6 thickneſſes.

The

The extent from	for Brick	to				Foot	
30	$\frac{1}{2}$		being			80	reduced to a Brick & half thick.
15	1	to	laid			160	
10	2		the	20		240	
7$\frac{1}{2}$		12	ſame			320	
6	2$\frac{1}{2}$		way		give	400	
5	3		from			480	

To reduce any Number of Feet, to Rods, Quarters, and Feet, at one ſitting.

The extent from 272 one quarter to 1, being laid the ſame way from the Numbers of Feet reaches to the Rods and quarters, and decimals over; then the ſame extent, laid the contrary way from the decimal Parts more than a Rod or quarters, give the Number of odd feet over.

Example, *in 2578 feet, how many Rod, Quarters and Feet?*

The extent from 272$\frac{1}{4}$ to 1 being laid the ſame way from 2578 gives firſt 9 Rod and a quarter, and the Decimal

Parts over are 22, then the ſame extent laid the contrary way from 22, gives 60 the odd feet over.

Then for the price of the whole or part; As $272\frac{1}{4}$ to 5 *l.* the rate of 1 Rod; ſo is 2578 feet to 47 *l.* 7 *s.* the price of the whole, or ſo is the odd 60 foot to 1 *l.* 2 *s.* the like for any other.

Uſe XVIII.

To Meaſure digging by ſolid Yards.

1. AS 1 to the breadth, ſo is the length to the ſuperficial Content in feet.

2. As 1 to the Content in Superficial feet, ſo is the depth to the content in Solid feet.

3. As 27 the feet in 1 Yard, to 1; ſo is the Content in Solid feet, to the Content in Yards,

Or ſhorter thus,

1. As 9 (the number of ſuperficial feet in a ſuperficial yard,) to the length in feet; ſo is the breadth in feet to the content in ſuperficial Yards.

2. As 3 (the depth of one yard) to the content in ſuperficial yards; ſo is the depth in feet, to the content in ſolid yards.

Example, *At* 20 *foot long*, 18 *foot broad, and* 8 *foot deep, how many yards of Earth?*

1. The Extent from 9 to 20, reaches from 18 to 40 the ſuperficial yards.

2. The extent from 3 to 40 reaches from 8 the depth to 106 $\frac{2}{3}$ (or 18 foot) the true content in yards or loads of Earth.

Uſe XIX.

Of Simple Intereſt.

1. HAving the Rate of Intereſt for 100 *l.* to find the Intereſt due for a greater or leſſer ſum for one year.

As 100 *l.* to 6 *l.* 7 *l.* or 8 *l.* the rate due for 100 *l.* in a year; ſo is any other ſum to its Intereſt due in 1 year.

Example, *As* 100 *to* 6, *ſo is* 124 *l.* *to* 7 *l.* 8 *s.* 9 *d.*

2. For Moneths reckon thus, if 6 be 12 Moneths (or 6 *l.* the Intereſt for 100 *l.* in 12 Moneths) then 9 is 18 Moneths, 12 is 24 Moneths, *&c.*

Thus the Intereſt of 30 *l.* in 18 moneths comes to 2 *l.* 14 *s.* for the extent from 100 to 9 reaches the ſame way from 30 to 2 *l.* 14 *s.*

3. To find Principal and Increaſe, or the preſent worth of any ſum for any time.

Count thus, The middle 1, a 100 *l.* and 6 tenths further for 106 the principle and increaſe of 100 *l.* in one year, 112 is for 2 years, 118 for 3 years, 124 for 4 years; thus every tenth is 2 moneths: ſo that for 7 years count thus, 7 times 12 is 84, whoſe half 42, or rather 142, is the Point for 7 years.

Example, *What is the increaſe and principal of* 30 l. *in* 5 *years and* ½?

The Extent from 100 to 133 laid the ſame way from 30, reaches to 39 *l.* 18 *s.* the increaſe and principal at Simple Intereſt.

But if the ſame extent be laid decreaſing from 30, it gives but 22 *l.* 11 *s.* being the preſent worth of 30 *l.* due 5 years and a half to come, at Simple Intereſt at the rate of 6 *l.* *per Cent. per Annum*.

4. For any number of dayes work thus,

1. As 365 the dayes in 1 year, is to

the rate of Intereſt due for 100 *l.* in 365 dayes, ſo is any other number of Dayes to the Intereſt due for 100 *l.* at the end of thoſe dayes.

2. As the Sum laſt found with 100 added to it, is to 100, ſo is the ſum propounded to his preſent worth; or if you turn the other way to his increaſe.

Example, *What is the preſent worth or increaſe of* 148 *l in* 5 *years, at* 8 per Cent?

1. As 365 to 8, ſo is 1825 (the Dayes in 5 years) to 40 *l.* the Intereſt due for 100 *l.* in 5 years at 8 *per Cent.*

2. As 140, (the Sum laſt found and 100 added) to 100, ſo is 148 the Money due at 5 years end, to 105,7142 the preſent worth.

Or if you turn the ſame extent the other way, to 207 *l.* 12 or 2 *s.* 5 *d.* the increaſe of 148 *l.* in 5 years time, at Simple Intereſt at 8 *per Cent.*

5. As for Equation of Payments in brief thus,

As the Sum of the preſent worths to pay preſently, to the Sum of the payments to be paid in time, ſo is 1 to a 4th number; from which number when 100 is ſubſtracted, remains the Intereſt of 1 *l.* for the time ſought, which divided by 00016438, the Quotient is the Anſwer.

Example, *If* 120l. *be agreed to be paid by* 40l. *at* 3 *Moneths end, at how many dayes end ought it to be paid at once to allow Intereſt for the preſent payment?*

The 3 preſent worths come to 116*l.* 521.

Then as 116,521 to 120, ſo is 1 to 102,98; from which taking 100, or 1 in the place of a 100, reſt 02,98, the Intereſt of 100*l.* the time required.

Laſtly, The extent from 00016438 to 1 laid the ſame way from 02,98, gives 181 $\frac{1}{3}$ the number of dayes required; wherein the 120*l.* ought to be paid all at once, to allow Intereſt at 6 *per Cent.* to both.

As for increaſe and preſent worths of Annuities at Simple Intereſt, I refer you to a Diſcourſe by it ſelf, with ample and exact Tables both for Simple and Compound Intereſt ready for the Preſs, by *I. B.* and *E. H.*

Uſe XX.

Of Compound Intereſt.

ANy Sum of Money put out for time, to receive at laſt Compound Intereſt, to find the increaſe of Uſe and Principle at the time limited in years only.

The extent from 100, to the Principle and the increaſe of 100 *l.* in one year, being repeated ſo many times from the Sum propounded, as there be years in the Queſtion; ſhall ſtay at the Anſwer to the Queſtion required.

Example , *What is the increaſe o 30 l. 10 s. put out for 5 years end, t receive Compound Intereſt at the end o 5 years, at* 6 l. per Cent per Annum

The exact extent from 100, to 106 being repeated 5 times from 30 *l.* 10 gives 46 *l.* 17 *s.*

2. *To do the like for Moneths,*

Let the ſpace between 100 and 106 on the Line of Numbers be divided into 12 equal parts; then thus,

The Extent from 100 to the Number of Moneths propounded, ſhall reach from the Sum to the Principal and Increaſe at the end of thoſe Moneths (or dayes) propounded.

Example, *What ſhall be the increaſe of* 125 *l. in* 9 *Moneths?*

The extent from 100 to 9 Moneths, being laid the ſame way from 125 l. gives 130 *l.* 11 *s.* 9 *d.*

3. To do the ſame when many years and Moneths are propounded, you muſt uſe a Scale of equal Parts whereby the Line of Numbers was made, and uſe it thus,

	Dayes Log.		Months Logar.		Years Logar.
1	,00006928	1	,00210882	1	,02530586
2	,00013857	2	,00421764	2	,05061173
3	,00020786	3	,00632646	3	,05791759
4	,00027713	4	,00843528	4	,10122346
5	,00034642	5	,01054411	5	12652932
6	,00041571	6	,01265293	6	,15183519
7	,00048498	7	,01476176	7	,17714105
8	,00055431	8	,01687057	8	.20244692
9	,00062360	9	,01897993	9	,22775279
10	,00069281	10	.02108822	10	,253 5865
11	,00076215	11	,02319704	11	,27836452
12	,00083141	12	,02530586	12	,30367038

Theſe Numbers, are the Logarithmes of 12 Dayes, 12 Moneths, and 12 Years, wherewith by Addition, or Multiplication, you may gain the Logarithme of any number of Years, Moneths and Dayes whatſoever, and the Sum when added and taken from the Scale of equal Parts, and laid from the Sum propounded, give the increaſe of the Principal and Intereſt in the time propounded at 6 *per Cent.*

Example, *If* 53 l. 5 s. *be put out to Uſe for* 15 *Years*, 9 *Moneths, and* 17 *Dayes, what is the increaſe?*

The Log. of 10 years is —,25305865
The Log. of 5 years is —,12652932
The Log. of 9 moneths is--,01897993
The Log. of 10 dayes is—-,0006928
The Log. of 7 dayes is—-,0004849
The Sum of the Log. is---,3997456

Being the Logarithme of 15 Years, 9 Moneths, and 17 Dayes.

This extent taken from the Scale of equal Parts and laid increaſing from 53 *l.* 5 *s.* gives 133--13 *s.* for the Principal and increaſe of 53 *l.* 5 *s.* in 15 Years 9 Moneths, and 17 Dayes, at 6 *pe Cent.* Compound Intereſt.

2. T

2. *To find the Decreaſe, Rebate, or preſent Worth of any Sum for any time at* 6 per Cent.

The operation is the ſame as for the increaſe, only to be laid backward or decreaſing from the Sum propounded, as the other was laid increaſing; as in the laſt Example, the extent of the Compaſſes taken, from 0 of equal parts to 39974, laid on the Line of Numbers from 53 *l.* 25 *s.* decreaſing, gives 21 *l.* 4 *s.* 6 *d.* the preſent worth of 53 *l.* 5 *s.* due 15 years, 9 moneths, and 17 dayes to come, at 6 *per Cent.* Compound Intereſt.

3. *To find the Increaſe, or preſent worths of Annuities at* 6 per Cent.

Firſt find what Sum of Money it is that hath the Intereſt thereof equal to the Annuity or half year, or quarterly Payment in a year, half year, or quarter; then by the former Rule, find the increaſe of that Sum, in the time propounded, and from the Sum ſubſtrac the firſt found Principal, and the remainder is the increaſe required.

Example, *Suppoſe the Annuity be* 10 *l* per Annum, *and continues for* 21 *year what is the increaſe at* 21 *years end, o the preſent worth in ready Money to bu to gain at* 6 per Cent.

Firſt, as 6 to 100, ſo is 10 to 16 666, for a Principal whoſe Intereſt i 10 *l. per Annum*.

Then the Logarithme of 1 year at *per Cent.* viz. 02530606 multiplied b 21 is 5314232; which taken from equ Parts, and laid from 166,666 increaſ ing, gives 566,666 for the Princip and Arrears in 21 years; from whi taking 166,666 remains 400 *l.* for t anſwer as Increaſing.

Then the ſame extent laid decreaſ from 400, gives 117 *l.* 12 *s.* the ſent worth of 10 *l.* *per Annum*, to continue 21 years to gain at 6 *per Cent.*

For other Rates make other Logarithmes, Tables for Years, Moneths and Dayes.

Uſe XXI.

Of the 30 and 40 Scales.

1. First it is a fixed Scale of 30 foot, divided to every foot and inch from 1 to 30.

2. The Scale to 40 foot is the like, where every foot is numbred, and every 2 Inches expreſſed, from 1 foot to 40 foot.

3. The 30 Scales are very fit in Architecture, for large or ſmall Draughts; as thus,

1. In large Draughts, let 30 repreſent half a Model, then every Foot is

one Minute, and every inch one twelfth part of a Minute.

2. In moderate Draughts, the whole Scale to 30 may repreſent 3 Models, then every foot is 6 Minutes, and every Inch is half a Minute.

3. In ſmall Draughts, let every foot repreſent 1 Model, then every Inch is 5 Minutes, for 12 times 5 is 60, the Minutes in 1 Model.

Thus the 30 Scales, may conveniently ſerve for greater, or leſſer Scales, by opening and ſhutting the Lines Sector-like.

4. The meaning of Four Terms in the following Uſes, uſed for brevity ſake, *viz.* *Lateral*, *Parallel*, *Common Line*, and *Neareſt diſtance.*

1. *Lateral*, is any diſtance or extent taken from the Center or beginning of any Line, along the Lin to the part required on one Line only

As the *Lateral* extent of 10 foot, i the meaſure from the Center to 10 foot on any one of the 30 Scales, or on th

Scal

Scale to 40, or in like manner on the Lines of Chords or Tangents, from the beginning to any Number or Point required.

2. *Parallel* is the extent taken acroſs from one Leg to another, as from 30, on one Leg, to 30 on the other.

Note, That in all Parallel extents you muſt ſet the Compaſs points in that Line only which runs up to the Center of the head Leg where the Braſs Pins are put, to ſet the Compaſs points into, (which you may call the *Common Line*.)

4. *Neareſt diſtance*, from a Point to a Line is only thus, ſet one point of the Compaſſes in the point given, and open and ſhut the other, being moved to and fro till it do juſt touch, or as it were cleave the Line, that extent I call the *Neareſt diſtance*.

5 *Any Line being given, and counted or called any Number of Parts, to add to it or take from it any parts in Proportion:*

one Minute, and every inch one twelfth part of a Minute.

2. In moderate Draughts, the whole Scale to 30 may repreſent 3 Models, then every foot is 6 Minutes, and every Inch is half a Minute.

3. In ſmall Draughts, let every foot repreſent 1 Model, then every Inch is 5 Minutes, for 12 times 5 is 60, the Minutes in 1 Model.

Thus the 30 Scales, may conveniently ſerve for greater, or leſſer Scales, by opening and ſhutting the Lines Sector-like.

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Scale to 40, or in like manner on the Lines of Chords or Tangents, from the beginning to any Number or Point required.

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Note, That in all Parallel extents you muſt ſet the Compaſs points in that Line only which runs up to the Center of the head Leg where the Braſs Pins are put, to ſet the Compaſs points into, (which you may call the *Common Line.*)

4. *Neareſt diſtance*, from a Point to a Line is only thus, ſet one point of the Compaſſes in the point given, and open and ſhut the other, being moved to and fro till it do juſt touch, or as it were cleave the Line, that extent I call the *Neareſt diſtance*.

5 *Any Line being given, and counted or called any Number of Parts, to add to it or take from it any parts in Proportion:*

Example, Let 6 inches be called 15 foot, and I would have 12 foot or 20 foot to the ſame proportion.

Take 6 inches between your Compaſſes, and make it a Parallel in 15 and 15 on the 30 Scales then the Parallel extent between 12 and 12 on the ſame 30 Scales, is a ſhorter Line in proportion, and the Parallel extent from 20 to 20, on the common Line of the ſame 30 Scales. is a greater Line in Proportion to the firſt 6 inches called 15 foot.

6. *To divide a Line into any Number of Parts.*

Take the given Line between your Compaſſes, make it a Parallel in the parts you would have it divided into, on the 30 Scales, then the Parallel extent between 1 and 1, ſhall divide the Line into the parts required.

Example, Let 5 inches be a Line I would divide into 15 parts, make it a

Parallel in 15 and 15 on 30 Scales, then Parallel 1 and 1, ſhall divide it into 15 parts, or if you make it a Parallel in 30, the double of 15, then take out Parallel 2 and 2, the double of 1, and that ſhall do better.

7. *Any two Lines given, to find their Proportion one to another in Numbers.*

Take the Lines ſeverally between your Compaſſes, and lay them latterally from the beginning of any Scale, and the Numbers in which they ſhall terminate, on the ſame Scale ſhall be their Proportion one to another.

But if the name or Number of parts of one Line be reſolved on, then thus,

Make that known Line a Parallel in the ſame like parts on the 30 Scales, then take the other Line between your Compaſſes, and carry it parallel, till it ſtay in like parts, on both 30 Scales, and that ſhall be the Name to the other Line in like proportional parts.

Example, Let 5 inches and 7 inches, be two Lines; let 5 be called 15, then what is 7?

Make 5 inches a parallel in 15 and 15 on the 30 Scales, then 7 inches ſhall ſtay parallelly only in 21 and 21; the Anſwer.

For by *Gunter's* Line, As 5 to 15, ſo is 7 to 21.

8 *To any two Lines given, to find a Third in continual Proportion* (Geometrical) *either increaſing or decreaſing.*

Lay both Lines latterally on both Legs, and note where they end, and when you increaſe, take the greateſt lateral Line, and make it a Parallel in the Terms of the leaſt, then the Parallel Terms of the greateſt, gives the lateral third Terme, in continual Proportion.

But when you would decreaſe in Proportion,

portion, then fit the lateral leſſer Term in the Parallel greater, then the Parallel Terms from the leaſt, ſhall give a lateral leſſer Term; which by the Line of Numbers is wrought thus: The extent from the firſt term to the ſecond, reaches the ſame way from the ſecond to the third.

Example, The extent from 3 to 5, reaches the ſame way from 5 to 8,333, a third in Proportion *Geometrical.*

9 *To divide one line like another, though it be greater or leſſer.*

Lay the whole divided Line laterally, and fit the Line to the divided parallelly in the end thereof: and note the parts to which it extends, (or lay the other common Line to the neereſt diſtance,) then every part of the divided Line laid laterally, and the neareſt diſtance from thence to the Common Line, taken Parallelly, ſhall divide the other Line according to the firſt Line.

10. *To find a mean Proportion between two Lines or Numbers.*

Open the 30 Scales to a right Angle, by making lateral 10 foot a parallel in 6 foot and 8 foot, or 21 foot 2 inches a paral. in 15 foot, then of the two Lines or Numbers find the ſum, the half ſum, difference, and ½ difference, then taking the half ſum between your Compaſſes, and ſetting 1 point in the half difference, counted on one 30 ſcale, extend the other point to the other 30 ſcale on the common Line, and there it ſhall give the mean proportional Number required.

Example.

If a piece of Timber be 6 inches thick, and 13 inches and ½ broad, what is the Square equal, which is the Geometrical mean Proportion between 6 and 13 ½, the Sum is 19 ½, the half ſum is $9\frac{3}{4}$, the difference is $7\frac{1}{2}$, the $\frac{1}{2}$ difference is $3\frac{3}{4}$.

Take $9\frac{3}{4}$ between your Compaſſes, and the 30 Scales ſet ſquare, put one point in $3\frac{3}{4}$ on one 30 ſcale, and the other Point applied to the common Line, gives 9 on the other 30 ſcale for a Geometrical mean required; for by *Gunter's* Line, the Middle between 6 and $13\frac{1}{2}$ is 9 the mean.

11 *To work the Rule of Three by the 30 Scales.*

As lateral ſecond, to parallel firſt;
So is parallel third, to lateral fourth.

Example, *If 5 foot of wood coſt 4s. what coſt 23 foot?*

Make lateral 4 *s.* a parallel in 5, then parallel 20 foot, gives lateral 16 *s.*

12 *To meaſure Superficial meaſure.*

1 The inches broad being given, to find the length of 1 foot.

As Lateral 12 to Parallel breadth: So is Parallel 12, to lateral length.

Example, At 8 inches broad, you ſhall find 18 inches long makes 1 foot.

2 *The breadth in inches and length in feet given, to find the Content.*

As Lateral length, to Parallel 12: So is Parallel breadth, to lateral Content.

Thus at 8 inches broad, and 10 foot long, you find 7 foot and $\frac{1}{2}$.

13. *To meaſure Solid meaſure by 30 Scales.*

1 To find how much long makes 1 foot: As lateral breadth, to parallel 12, So is parallel depth to lateral fourth. Again, As lateral 12, to parallel fourth, ſo is parallel 12, to lateral length.

Example, at 9 and 14 you find 13 inches $\frac{3}{4}$ in length, to make 1 foot of Timber.

2. *The*

2. *The ſides of the ſquares and length given, to find the Content in feet.*

As lateral breadth in inches, to parallel 12, ſo is parallel depth in inches to lateral fourth.

2 As lateral fourth to parallel 12, ſo is parallel length in feet to lateral Content.

14. *To divide a Circle into any Parts.*

Make the Semidiameter a Parallel in 6 and 6, (at 15 foot) then the Point required taken Parallelly, ſhall divide the Circle into the parts required.

Example, Would you have a Circle divided into ten parts?

Make the Semidiameter a Parallel in 6 and 6, the Parallel extent from the points of the ſmall Figures at 10 and 16, ſhall divide the Circle into 10 parts required: and ſo for any other.

15 *At any breadth of a building to find the Rafters and Perpendiculars lenghts, at true pitch, by inſpection only.*

Seek for the breadth of the Frame on the 40 Scale, and juſt againſt it on the 30 Scale, is the Rafters length required.

Then count the Rafters length on the 40 Scale, and juſt againſt it on the 30 Scale, is the Perpendicular Altitude of the Gable end, above the raiſing piece.

Example, At 24 foot wide, you find 18 foot for the Rafters length, and at 18 foot for Rafter I find 13 foot and 6 inches for Perpendicular.

16 *The half breadth of the Frame, and the Perpendicular height reſolved on, to find Rafters, Hips and Sleepers length, and Angles at Foot and Top.*

Open the 30 Scales to a Right Angle

by 5,3, and 4, or 10, 8 and 6, or 21 *f.* 2 *inches*, and 15 *f.*

Then count the half width of the Frame on one leg, and the Perpendicular height reſolved on, on the other leg, and take that extent between your Compaſſes, and lay it from the Center, and it ſhall give the Rafters length required.

Then for the Hips length, the Parallel extent from the half width to the Rafters length, laid latterally from the Center, gives the Hips length.

For the Diagonal Line, count the half width of the Frame on each leg, and the parallel extent between ſhall be the lateral half Diagonal, or the meaſure from the corner to the King poſt; for the Angles, a rule laid to thoſe points on the 30 ſcales, and a bevel ſet to that rule and the common line on the 30 ſcales, give all the Angles required.

Example, Let the frames width be 24 foot, and the perpendicular reſolved on 13 foot ½, then the rafters length will be 18 foot, the hips length 21 foot 7 inches and $\frac{1}{4}$, the half Diagonal near 17 foot, the rafters angles at foot 48 gr. 10 m. at foot 41 gr. 50 m. the hips angles at foot 38, gr. 22 m. at top 51 gr. 38 m. the outſide of the hip at true pitch 116 gr. $\frac{1}{4}$. The Angles being the ſame in all ſquare frames at true pitch, but found as above at any pitch. As for bevel and taper frames they are more difficult and require a ſcheam to make it plain, for the which I refer you to the Appendix to Scamoſſy, where it is done at large.

17. *To meaſure any Angle by the Chords.*

Apply your rule to any angle of a building, or ſet the rule to any angle you pleaſe, then the parallel extent from 15 to 15, meaſured laterally on the line of Chords, gives the angle the line of 30 ſcales ſtand at, the rule being commonly 2 degrees leſs; for if you meaſure from the inſide to the inſide juſt againſt 15, it ſheweth on the chords the angle that the rule it ſelf ſtandeth at.

On the contrary, to ſet the 30 ſcales, or the rule to any angle.

Take out the lateral chord of the angle you would have, and make it a parallel in 15 and 15, or juſt againſt 15 on the inſide, and it ſets the lines or the rule to the angle required.

Uſe XXI.

The Uſe of the Quadrant ſide.

1. TO rectify the rule to 60 degrees, make the lines on the head leg, and the hour line on the moving leg,

leg ; to become ſtrait and whole. which otherwiſe ſeems broken, and it is near to 60 degrees.

Or with Compaſſes, take any extent from the rectiſying point on the head leg, to any degree of altitude on the ſame leg, and note it ; then turn the ſame extent, to the common line on the moving leg, and then remove the compaſs point from the rectiſying point, to the common line on the head leg, and if it hit the firſt meaſured point, it is exact, elſe not,

Example, Suppoſe I take the extent from the rectifying point on the head, (where the hour and azimuth line, and the line of ſines or altitudes meet) to the center where the thred is faſtened, and lay the ſame extent from the rectiſying point on the Azimuth line, it ſhall reach to 43 gr. 20 m. of azimuth (or at leaſt to a point, on which the thred being laid cuts 30 degrees) and the ſame extent laid from that point of 43 gr. 20 m. on the azimuth line, does reach juſt to the center again, ard ſo make an equal ſided triangle; and the rule is ſet to 60 degrees, and fit for obſervation.

Or if you have a third piece, with two tennons at each end; then put each tennon in his right mortice hole, at the end of the rule, and then the joynts pulled cloſe, the rule is ſet to the exact angle of 60 degrees.

2. *To try if any thing be level, or to draw a level line.*

On the thred hang a plummet, and apply the moving leg to the object, and if the thred cut 60 gr. 0 m. on the line of degrees it is level, or elſe not; and how many degrees and minutes out of level, and which way, the thred on the degrees ſheweth.

Then on a plain along by the moveing leg, you may draw a level line.

5. *To find the Suns riſing and ſettin declination, and the like.*

Open the rule to 60 degrees by (pu ing in the looſe piece) the lines compaſſes , then draw the thred in center ſtrait over the day of the mon then in the line of deg. it ſheweth Suns declination, counting from 60 0 m. to the right hand for north declin or to the left hand for ſouth declinatio according to the time of the year, *vi* north declination in ſummer, and ſou declination in winter, and in the ho line it cuts the ſuns riſing and ſettin and his place and right aſcenſion, in lines of the ſuns place and right aſce ſion, if you have them on the rule.

Example on the third of April,

The rule ſet to 60 degrees, and thred in the center drawn ſtrait a laid over the third of *April*, ſhewe deg. 15 minutes north declination on he right hand of 60 gr. 0 m. alſo in the our line, it ſheweth 5 gr. 13 m. the time of ſuns riſing, or 6 gr. 47 m. the time of n ſetting, according to the progreſs of he fore-noon or after-noon hours.

And in the lines of the ſuns place 4 gr. in *Aries*, and in the line of right ſcenſion, 1 hour 28 minutes, and he like for any day in the year.

6. *To find the hour of the day.*

Rectiſy the rule by ſetting it to 60 degrees, then find the ſuns altitude, by obſervation, and with compaſſes take it from the particular ſcale of altitudes, and lay the thred to the day of the month (or declination of the ſun,) then carry the one point of the compaſſes along in the hour line, till the other foot being turned about, will but juſt touch the thred at the neareſt diſtance, then the compaſs point in the hour line, ſhall ſhew the hour and minute required.

Exam-

3. *To try if any thing be upright, or to draw a perpendicular line.*

The plummet hanging on the thred, apply the head leg to the wall, or poaſt to be examined, and if the thred cut the rectifying point or ſtroke at 90 on the head, when workman like held the plain or poaſt is upright, and along by the head leg, may you draw on the plain a perpendicular line.

4. *To find an Altitude by the degrees.*

Firſt ſet the rule to 60 degrees, then a plummet faſtened on the end of the thred, look up to the object, by the upper or lower leg, according as the object is high or low, then the thred playing evenly by the moving leg, ſhall on the degrees ſhew the angle of Altitude required.

Note, that if the object be low (by your being far from it) then to look along by the moving leg is moſt convenient, and then you muſt count the degrees of altitude from 60 gr. 0 m. the moving leg towards the head, with 10,20,30 at the head.

But if the Altitude be above 30 degrees, you muſt look along by the head leg, and then the figure ſtanding before the lines being greater than the other, and numbred from 25, or 30, to 90 at the head; Note, that when the Sun ſhines bright, the ſhadow of the inſide of the legs will plainly be ſeen on the head next the inſide, to ſet the rule exactly, and the leaning your hand againſt ſome ſteady thing, will eaſe and help you to be exact in the doing of this which is the foundation or baſis of many propoſitions; a heavy plummet is much better than a light one, and moves on the leaſt variation more ſteadily.

5. *To find the Suns riſing and ſetti*
declination, and the like.

Open the rule to 60 degrees by (p
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compaſſes , then draw the thred int
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And in the lines of the ſuns place 4 gr. in *Aries*, and in the line of right ſcenſion, 1 hour 28 minutes, and he like for any day in the year.

6. *To find the hour of the day.*

Rectiſy the rule by ſetting it to 60 degrees, then find the ſuns altitude, by obſervation, and with compaſſes take it from the particular ſcale of altitudes, and lay the thred to the day of the month (or declination of the ſun,) then carry the one point of the compaſſes along in the hour line, till the other foot being turned about, will but juſt touch the thred at the neareſt diſtance, then the compaſs point in the hour line, ſhall ſhew the hour and minute required.

Example , April 3, *at* 20 *degr* *high in the fore-noon , you ſhall find t* *hour to be* 23 *minutes paſt* 7.

But at the ſame height on the firſt *October* , you ſhall find 4 minutes af 9 in the forenoon, or 56 paſt 2 in t afternoon, in latitude of 51 gr. 30 being the latitude of *London*.

Note, That the contrary to this, fin the ſuns altitude at any hour, at a time of the year; for if the thred laid to the declination, and the near diſtance taken from the hour to t thred (in the right line) and meaſur in the particular ſcale of altitudes, gives the ſuns altitude required at th hour or minute.

7. *To find the Sun Azimuth.*

Take the Suns declination from t particular ſcale between your Co paſſes, and lay the thred to the ſu

altitu

ltitude, counted on the degrees from 60,0 toward the end, and ſometimes eyond on the looſe piece, and there eep it fixed; then carry the Compaſſes on the north ſide in north declinations, and on the ſouth ſide of the thred n ſouth declinations, that is to ſay, on he right ſide of the thred in ſummer, nd on the left ſide in winter, in the Azimuth line, ſo as the other point urned about, will but juſt touch the hred at the neareſt diſtance, then the oint fixed in the Azimuth line, ſhall hew the ſuns Azimuth from the ſouth equired.

Example, At 15 deg. of declination and 20 deg. high, take 15 deg. from he particular ſcale of altitudes, between your compaſſes, then the thred id to 20 deg. counting from 60,0 oward the end ſo keep it.

Then the neareſt diſtance from the Azimuth line to the thred ſheweth 89 deg. the compaſſes being carried on the ight ſide of the thred, but being ſet

on the left ſide of the thred , at th
neareſt diſtance in the Azimuth line, i
ſheweth 26 deg. from the ſouth, for th
ſuns Azimuth in the winter at 15 deg
of declination, and 20 deg. of altitude.

Note, *In the equinoctial, when the ſu hath no declination, then there is 0 to tak between your compaſſes ; then if you la the thred to the ſuns altitude in the deg being counted from 60,0 in the Azimut line , it ſhall ſhew the Azimuth requir*

Example, In latitude 51 deg. 30 m at 10 deg. high, the Azimuth is 77 deg 15 m. from ſouth, at 20 deg. high 6 deg. 45 m. at 30 deg. high , 43 de 15 m. at 38 deg. 30 m. high, juſt ſouth

8. *To draw a Meridian line.*

On any flat plain, when the ſun ſhin eth , draw the line on the plain, mad by the ſhadow of a thred and plumme hold up , and ſtrait way (or rather the ſame inſtant let another) take th ſuns altitude, and with that find th

ſu

ſuns azimuth, then making a center in the ſhadow line, lay off from thence the right way, the angle of the ſuns azimuth found, and draw that line, (the ſuns motion, and time of the day conſidered, will certainly inform you which way the ſouth is from the preſent ſhadow line) that line I ſay workman like performed and drawn, ſhall be the true Meridian line required.

9. *To find the latitude of a place.*

Juſt at noon, when the ſun is in the meridian, find the ſuns altitude, and ſet it down, and ſubſtract it from 90 to find its complement to 90.

Then in winter ſubſtract the ſuns preſent declination from the complement of the ſuns altitude, and the remainder is the latitude, but in ſummer add the ſuns co-altitude and declination, and the ſum ſhall be the latitude.

Example, On *May* the 10th. the meridian altitude is 58 deg. 37. m.

whoſe complement to 90 deg. is 31 deg. 23 m. the declination the ſame day is 20 deg. 07 m. the ſum after addition i 51 deg. 30. m. the latitude of *London*.

Or again, in winter or ſouth declination, on *November* the 10th. the ſuns declination is 19 deg. 53 m. the meridian altitude is 18 deg. 37 m. whoſe complement is 71 deg 23 m. from which taking 19 deg. 53 m. remains 51 deg. 30 m.

There be many other cautions, for ſmall and great latitudes to be conſidered, which I ſhall not here ſpeak to, but refer them to the judgment and reaſon of the practitioner.

10. *To find the hour in other latitude the general ſcale of altitudes, an line of degrees.*

Take the lateral ſine of the ſum o the ſuns preſent altitude, and his depreſſion at 6 in winter, and make it

paral

parallel ſine in the co-altitude, and lay the thred to the neareſt diſtance, then the neareſt diſtance from the ſecant of the ſuns declination, beyond 90 to the thred, taken parallelly between your compaſſes, ſet one point in 90, and the thred laid to the neareſt diſtance, ſhall ſhew the hour from 6 in the deg. counting 15 deg. for an hour.

Example, *The latitude* 50 *degrees, the declination* 20 *degrees ſouth, the altitude* 10 *degrees, what is the hour?*

Lay the thred to 70 deg. being 20 deg. from the 90 at the head, the ſuns declination ſo counted, then the neareſt diſtance from the ſine of 50 to the thred, is the ſuns altitude at 6 in ſummer, or his depreſſion in winter, being in this example 15 deg. 16 m. and to be laid in ſummer from the center downwards, to be ſubſtracted from the ſuns preſent altitude, but in winter to be laid upwards beyond the center, to be added to the ſuns preſent altitude; as in this preſent example reaching to 52 $\frac{1}{2}$ on the ſmall line of ſines beyond the center; then the lateral exten from thence to the ſine of 10 deg. on the general ſcale, being made a pa rallel in 40, the co-latitude, lay th thred to the neareſt diſtance, and ſo keep it; then take the neareſt diſtanc from the ſecant of the declination (be yond 90 on the head) to the thred, an make it a parallel ſine of 90, and la the thred to the neareſt diſtance, an on the deg. it ſhall ſhew the hour an minute required, *viz.* 7 m. after 9 i the fore-noon, or two hours and 5 m. in the after noon.

But in ſummer, at the ſame declina tion and altitude of 10 deg. you ſhal find the hour to be 5 h. and 27 m. i the morning, or 6,33 in the after noon; 90 at head being alwayes 6, an 0,60 on the looſe piece alwayes 12 which work in ſhort is worded thus.

As the latteral ſine of the ſum of the ſuns preſent altitude and altitude at 6 in winter, or difference between the ſuns preſent altitude and altitude at 6 in ſummer, taken onely on ſines:

To the parallel co-ſine of the latitude, ſo is the parallel ſecant of the declination, made a parallel ſine of 90, to the hour on the deg. counting from 90, as before in the precept.

More of this, with many other wayes may you find in the Triangular Quadrant, Chap. 15.

11. *To find the Suns Azimuth generally.*

Firſt, to find the azimuth in the equinoctial, take the tangent of the co-altitude from 60,0 on the moving leg toward the end on the degrees; make it a parallel ſine of 90, and lay the thred to the neareſt diſtance, and ſo keep it, by noteing the place where it lyes; then from the ſame line take the tangent of the ſuns preſent altitude, and carry it parallel till it ſtay in the ſines at the neareſt diſtance to the thred, and that ſhall be the ſine of the azimuth required, at that altitude in the equinoctial: or thus without tangents,

Take the lateral ſine of the ſuns altitude, make it a parallel in the co-ſine of the latitude, and lay the thred to the neareſt diſtance, and ſo keep it; then the neareſt diſtance from the ſine of the latitude to the thred, being ſet in the co-ſine of the ſuns altitude, and the thred laid again to the neareſt diſtance, give the ſuns azimuth from ſouth as numbered, or the azimuth from eaſt or weſt, counting on the head:

Example, Latitude 51 deg. 30 m. Suns altitude 30 deg. the ſuns azimuth from ſouth is 43 deg 15 m. or 46 deg. 45 m. from eaſt.

But for all other times, you muſt firſt find the ſuns altitude at eaſt or weſt thus: Take the lateral ſine of the ſuns declination, make it a parallel in the ſine of the latitude, and lay the thred to the neareſt diſtance, and on the deg. the the thred gives the altitude at eaſt or weſt: This altitude at eaſt, you muſt ſubſtract on a line of natural ſines, from the ſine of the ſuns preſent altitude, or take the leſſer from the greater, to find a difference with compaſſes on the general ſcale: This reſidue between your compaſſes, ſet one foot in the coſine of the latitude, and lay the thred to the neareſt diſtance, then take the neareſt diſtance from the ſine of the latitude to the thred, and ſetting one point in the co-latitude, and lay the thred to the neareſt diſtance again, and and in the deg. it gives the ſuns azimuth from ſouth required,

But in winter work thus.

Firſt find the ſuns amplitude, thus, Take the ſine of the ſuns declination, ſet one point in the co-latitude, and lay the thred to the neareſt diſtance, and in the deg. it gives the ſuns amplitude required; which remember: Then take the ſine of the ſuns altitude, and ſetting one point in the co-ſine of the latitude, lay the thred to the neareſt diſtance, and take the neareſt diſtance from the ſine of the latitude, to which you muſt add the ſine of the amplitude firſt found : Then this whole added diſtance between your compaſſes, ſet one point in the co-altitude and lay the the thred to the neareſt diſtance, and on the deg. it gives the azimuth from ſouth, or eaſt or weſt, as you reckon it; from 0,60 on the looſe piece, or 90 at head.

An Example for ſummer time is this, The ſuns declination 13 deg. 15 m. the altitude 37 deg. 27 m. the altitude at eaſt 17. deg. 1 m. the ſuns azimuth from ſouth will be 60 deg latitude 51 deg. 30 m. And at the ſame declination in winter the Suns amplitude will be found to be 21 deg. 36 m. then at 5 deg 55 m. of Suns altitude, you ſhall find 60 deg. of azimuth in latitude 51 deg. 30 m.

Uſe XXII.

The Uſe of the Almanack.

THe *Almanack* hath uſually 10 ranks of figures and letters whoſe names are expreſt on the leaſt ends of thoſe ranks, *viz.* months in 2 ranks, or 1 large one; days in 5 ranks for 5 times 7 dayes, then the week days, with *S.* for ſunday, and the reſt in order: Then a rank of leap years, and laſtly a rank of epacts right under, being the epact fit for thoſe leap years, and ſometimes the dominical letters, being known by their names on the left end, *viz.* W. D for week dayes, L. Y. leap years, D. L. for dominical letters and epacts, whoſe uſe is thus.

1. *To find on what day of the week, the firſt day of* March *will fall on any year.*

If the year propounded be a leap year, and expreſſed in the Almanack, then the letter right over for week days ſheweth it : *Example*, For the year 1676, expreſt there, and the letter right over it is W. for Wedneſday. But if it be any other year between a leap year, then thus, ſeek the leap year laſt paſt before the year propounded, and then as before the week day right over is the firſt of *March*, for that leap year; then the day changing ſucceſſively as the week days proceed, as thus : If the firſt of *March* on a leap year be Monday, the firſt of *March* the next year following, will be Tueſday, the next Wedneſday, the next Thurſday and then the other next ſucceeding leap year is expreſſed in the Almanack; thus a right counting from the laſt leap year anſwers the queſtion.

Example, Suppoſe I would know on what day of the week the firſt of *March* ſhall be in 1677: Firſt in 1676, the Almanack ſheweth Wedneſday, then in 1677 it will be Thurſday, in 1678 Fryday, in 1679, Saturday, and in 1680, Monday again, as the Almanack ſheweth, and the like for ever.

2. *Having the week day for* March 1 *for any year, to find what day of the week any day in any month ſhall be that year.*

In 1677, on *June* 4th. what day of the week is it, *March* 1 being Thurſday? Firſt according to the Old Roman account. *March* is the firſt month, then is *June* the 4th. then in the rank of the months find 4, which is *June* the 4th. month, then all the days right un- 4, in the year 1667, are Thurſdays, *viz.* the 7th. 14th. 21th. and 28th days, then if the 7th be Thurſday, the 8th. is Fryday, the 9th. is Saturday, &c. So that the 4th. day, the 11th. 18th.

and 25th. days of *June*, in 1667 are Mondays.

To find the Epact any Year, and then by that to find the Moons Age to a day.

Firſt in every Leap-year for 28 years, the Epact is expreſſed right under it, as right under 1676 is 25, the Epact that year; for the other three years between, add 1, 2, or 3 elevens, and the ſum if under 30 is the Epact required; *Example*, in 1678, what is the Epact? In 1676 the Leap-year it is 25, then 1678 being 2 years after, I add 2 elevens, *viz.* 22 to 25, the ſum is 47, and being above 30. I take away 30 and the remainder, 17 is the Epact for 1678. Note, that the Dominical or Sunday Letter, changeth *January* 1, but the week day and the Epact, on the firſt of *March*.

Then to find the Moons Age, add the Epact, the Month, and the day of the Month in one ſum, and that ſum if under 30, is the day of the Moons Age; ſo alſo is the remainder, when 29 or 30 is ſubſtracted; ſubſtract 30, when the month hath 31 dayes or but 29, having but 30 days: *Example*, In 1676, on the 10th. of *May*, what is the Moons Age? The Epact is 25, the month 3, the day 10, the ſum 38; from which when 30 is ſubſtracted, remains 8, the Moons Age to day.

Uſe XXIII.

The Uſe of the General Scale of Sines in part.

1 *The ſine of an Arch or Angle given, to find Radius.*

TAke the ſine given betwixt your Compaſſes, and ſet one point in the ſame ſine on the general Scale, and with the other lay the thred to the neareſt diſtance, and there keep it, then the neareſt diſtance from 90 to the Thred is the *Radius* required to be found.

2. *Radius, or any known Sine given, to find any other.*

Take Radius or the ſine given, between your Compaſſes, and ſetting one point in Radius or Sine 90, if that be given, or in the ſine given on the general Scale, and with the other point lay the thred to the neareſt diſtance, then the neareſt diſtance from any ſine you would have to the thred, ſhall be the ſine required; or if you have any unknown ſine, carry it nearer or further from the center, till the other point turned about will but juſt touch the thred, then the fixed point is the ſine of the meaſure required to that Radius.

3. *The Radius being given, to find any Tangent or Secant to the ſame Radius, by the general Scale of Sines onely.*

Take the given Radius between your Compaſſes, and ſet one point in the ſine complement of the Tangent required, and lay the Thred to the neareſt diſtance, and ſo keep it; then the neareſt diſtance from the ſine of the Tangent required, ſhall be the Tangent, and the neareſt diſtance from ſine 90, to the Thred, ſhall always be the Secant required.

4. *Any Tangent or Secant given, to find Radius, and then any other Tangent or Secant required.*

If a Tangent be given, make it a parallel in the ſine thereof, and lay the Thred, then the parallel co-ſine is Radius.

If a Secant be given, make it a parallel always in 90, and lay the Thred (to the neareſt diſtance as always before and after) then the parallel coſine ſhall be Radius; then having the Radius, make uſe of the laſt Rule, to find any thing required.

5. *To lay down any Chord, to any Radius leſs then the ſine 30 on the general Scale.*

Take the given Radius, make it a parallel in 30, and lay the Thred, an being laid, note what deg it cuts, fo the more ready ſetting it again, if yo uſe it often; then the neareſt diſtanc from the ſine of half the angle required ſhall be the Chord required.

6. *To lay down any Chord, or Angl to the Radius of the ſine 90, on the general Scale.*

Take the whole lateral length, fro

th

the Center to 90 on the general Scale, and lay it down on any line from the point from whence you would raiſe the angle, then take the ſine of the angle required from the Center downward laterally, ſetting one Compaſs point in the point made in the end of the line, and with the other draw the touch of an arch; then I ſay a line drawn from the deſigned Center, to the touch of an arch, ſhall be the exact angle required.

Note, If the whole length of the line be too large, you may uſe any leſs Radius to your mind parallelly; by ſitting the Radius in ſine 90, and laying the thred, then the neareſt diſtance from the ſine required, ſhall be the ſine to draw the touch of an arch withal.

7. *To lay down readily any Sine, Tangent, or Secant, to two Radiuſſes, viz. The greater on the general Scale, and moving Leg, the leſſer on the looſe piece, and beyond the Center.*

Seek the ſine required on the genera Scale from the Center downward, an that ſhall be the Sine, for the Tangent count from 0,60 on the movin Leg along the deg. and that extent taken with Compaſſes from the Center pin at 60,0 ſhall be the Tange required.

Then for the Secant the exte from the Tangent to the Center, ſhal be the Secant required

If the Radius of the general Scale b too large, then the leſſer line of ſine beyond the Center, on the head Leg is a line of ſines, and the proportiona Tangents is the very deg. on the looſ piece, counting from 60, toward th moving Leg, and the Secants, are, th meaſure from the Tangents to th Center.

Alſo, *Note*, That the leſſer Radiu is juſt one third part of the greater, ſ that if the greater Radius proceed no far enough, then take the ſmaller Radius, and turn the Compaſſes 3 time

an

and you have your want ſupplied; but more ample and plain diſcourſe, will you find in the Book, called the *Triangular Quadrant*, of theſe matters.

Uſe XXIV.

To find the Declination of any plain.

1 REctify the Triangular Quadrant, by putting in the looſe piece; then the Sun ſhining at 12 a Clock, apply the head Leg to the Wall, holding the Rule as level as you can; then hold up a Thred and Plummet, till it playing even and ſteady, the ſhadow of the Thred cut the Center where the Thred is faſtened, and alſo on the deg. on the head Leg, or looſe piece.

And Note, That that deg. is the declination required, as it is numbred from 0,60 on the looſe piece; for i the ſhadow fall on the left hand of 0,60 on the looſe piece, then the Declination is Eaſt-ward, if it fall on the right hand, then the plain declineth Weſt-ward, ſo many deg. as the ſhadow of the Thred ſheweth on the Inſtrument.

2. Another time alſo, to come by the Declination eaſily, is to wait till the ſhadow of the Thred, held as before, fall juſt on 0,60 on the looſe piece, that is when the Sun is juſt againſt the plain, then, by the Suns altitude taken at the ſame time, find the Suns azimuth, and that ſhall be the plains true declination required, according as the azimuth is, either Eaſt-ward or Weſt-ward, in the fore-noon or afternoon of the day.

3. If either of theſe times are too particular, or prove inconvenient, then thus at any time, except too near noon; apply the head Leg to the Wall, and hold up a Thred and Plummet, and

find

nd what Angle the ſhadow makes, nd on which ſide of 0,60 on the looſe iece, falling on the left hand paſt, nd the right hand want: which ſet own in Ink or Chalk, then preſently, s ſoon as poſſible, get the Suns altitude, and ſet that down alſo, with the ay of the month and time of the day, ither in the fore-noon or after-noon, n this manner as followeth in the Exmple.

On the 12th. of *May* I come to a Wall, and applying my Rule, and olding up a Thred and Plummet, the

May 12. 1674.	Forenoon.	
	gr.	m.
Suns Altitude	35	20
Shadow want	40	00
Azimuth want	78	30
South Eaſt	38	30

ſhadow falls on 40 degrees want, and t the ſame time obſerving the altitude

of the Sun, it is 35 deg. 20 m. then th
Suns azimuth, at that altitude, will b
found to be 78 deg. 30 m. wanting o
coming to South, as the ſhadow wante
of coming to the Pole of the plain
Then always obſerve to ſubſtract th
leſſer from the greater, the lines bein
alike, *viz.* both want, or both paſt
and the remainder 38 deg 30 m. is th
declination South Eaſt, becauſe th
Sun would come to the pole or meridia
of the plain before it comes to th
South, the meridian of the place.

Example the ſecond, Suppoſe th
to the ſame plain, I ſhould in the after
noon apply the Inſtruments, and th
ſhadow ſhould fall on 70 deg. pa

May 12, 1674. After-noon.

	gr.	m.
Suns Altitude	56	20
Shadow paſt	70	00
Azimuth paſt	31	30
South Eaſt	38	30

o de

deg. 60 m. and the Suns altitude the ſame time, 56 deg. 20 m. than the Suns azimuth at that altitude will be 31 deg. 30 m, paſt the South: here alſo the ſignes being alike, *viz.* both paſt, uſe Subſtraction, and the remainder 38 deg. 30 m. is the declination South Eaſt; becauſe it comes to the meridian of the plain, before it comes to the meridian of the place, *viz.* 12 a Clock.

3. A third Example, Suppoſe I had come to the ſame plain about 10 a Clock, and found the Sun to be paſt the plain 10 deg, and the altitude 37 gr.

May 12, 1674 Fore-noon.

	gr.	m.
Suns Altitude	57	0
Paſt plain	10	0
Azimuth want	28	30
South Eaſt	38	30

then the azimuth found at that altitude, will be 28 deg. 30 m, want of South;

becauſe before noon: Here the ſignes being unlike, *viz.* one want, the other paſt, uſe addition, and the Sum is 38 deg. 30 m. the declination South Eaſt, becauſe it comes to the meridian of the plain, before it comes to the meridian of the place South.

4. A fourth Example ſhall be of a North Weſt plain, As ſuppoſe at 6, or thereabout, after-noon, I come to a plain on the ſame 12th. of *May*,

May 12 1674. After-noon.

	gr.	m.
Suns Altitude	15	0
Shadow want	20	00
Azimuth Sun paſt	104	20
	124	20
ſ	180	00
	055	40

674. and applying the Rule to th
Wall, the ſhadow of the Thred cu
20 deg

20 deg. wanting of the 0 deg. 60 m. and at the ſame time the Suns altitude is 15 deg. then the azimuth will be 104 deg. 20 m. paſt the South; then after the addition, becauſe unlike ſignes, the Sum is 124 deg. 20 m. whoſe complement to 180, is 55,40 the declination North Weſt; becauſe the Sun in the South, cannot look or ſhine on ſuch a plain.

One of theſe four ways, will fit all Caſes can come, but for further confirmation, conſult the Tryangular Quadrant.

Note, *That a good Box and Needle, will do this work more eaſily, and be a good guide in the operation, if it be not attracted and drawn aſide by any magnetical property; yet the way by the Sun is the certaineſt and moſt artificial.*

Uſe XXV.

To find the Requiſites for Erect Decliners, as Subſtile, Stile, 12 *and* 6, *and the Inclination of Meridians, for that Latitude the Rule is drawn or made for,* viz. 51 *deg.* 30 *m. in the Figure.*

1. *To find the Subſtile from* 12.

LAy the Thred to the complement of the declination of th plain, counted on the azimuth line and on the line of deg. it gives the ſubſtile, counting from 60 deg. 0 m.

Example, Let a plain decline 20 deg the Thred laid to 70 deg. on the azimuth line, being the complement t 20 deg. ſhall cut on the line of de

15 de

15 deg 12 m. the ſubſtile from 12 deg. required.

2. *To find the Stiles height above the Subſtile.*

Take the diſtance on the azimuth line, between 90 deg. and the plaines declination, and meaſure it on the particular ſcale of ſines, from the beginning, and it ſhall reach to the Stiles height required.

Example, The extent from 90, deg to 20, gr. on the azimuth line, being laid from 0, on the particular ſcale of ſines gives 35 deg. 45 m. the ſtiles elevation above the ſubſtile required.

3. *To find the Angle between* 12 *and* 6.

Take the plains declination from the particular Scale of ſines (leſs by the ſine of the plains declination, to a Radius equal to the firſt 45 m. of the firſt

deg.

deg. on the ſame particular Sca'e of ſines) and lay it from 90 on the azimuth Scale , and to the compaſs point lay the thred ; then on the line of deg. the thred ſhall give the complement of 6 from 12, counting from 60,0 toward the looſe ipiece, or the Angle it ſelf counting 60,0 for 90, 50, for 80, *&c.*

Example, *For* 20 *deg. of declination.*

The extent from near 20 m. on the particular ſcale, to 20 deg. laid from 90 deg. on the azimuth line, and the Thred to that point, and drawn ſtreight on the deg. gives 23 deg. 18 m. or 66 deg. 42 m. the angle of 12 and 6 required.

4. *To find the Inclination of Meridians.*

Count the ſubtile on the particular ſcale of altitudes , and take the extent from 0 deg. to the ſame between your compaſſes , and meaſure it from 90 on the azimuth line, and it ſhall give the complement of the inclination of meridians.

Example, The ſubſtile for 20 deg. of declination, is 15 deg. 12 m. which taken from the particular ſcale of ſines, and meaſured on the azimuth line from 90 deg. gives 65 deg. 10 m. whoſe complement 24 deg. 50 m. is the inclination of meridians required; for a plain whoſe declination is 20 deg.

Thus you may find the requiſites for any declining plain, for that latitude the the Rule is made for, or by the general Scale of Sines, for any latitude in this manner.

Subſtile	*As* — *Cotang. lat.*	38	30
	To = *Sine* 90	90	00
	So = *Sine Declin.*	20	00
	To — *Tang. of Subſtile*	15	12

Stile	*As* —	*Coſine lat.*	38	28
	To =	*Sine* 90,	90	00
	So =	*Coſine Decly.*	70	00
	To —	*Sine Stile*	35	46

12 and 6.	*As* —	*Sine Declin.*	20	00
	To	*Sine* 90,	90	00
	So =	*Tang. lat.*	51	32
	To —	*Cotang.* 6 *and* 12	23	18

Incli. meri.	*As* —	*Tang Declin.*	20	00
	To =	*Sine lat.*	51	32
	So =	*Sine* 90	90	00
	To —	*Tang. In. M.*	24	56

More ways to find theſe, are ſet forth in the Triangular Quadrant.

The manner of working the laſt, and by conſequence all the reſt. by the general Quadrant or Sines, is thus:

Take the lateral Tangent of the declination 20 deg. counted from the

Center at 60 deg. 0 m. on the moving Leg toward the end, between your Compaſſes, and make it a parallel ſine of 51 deg. 32 m. in the geueral Scale of Sines, by putting one point in 51 deg. 32 m. and with the other lay the Thred to the neareſt diſtance, and ſo keep it: Then the parallel ſine of 90 taken by putting one pointin 90 deg. and open the other till it will but touch the Thred at the neareſt diſtance; then this extent meaſured laterally, from 60 deg. 0 m. on the tangent line, will reach to 24 and 56 the Inclination of Meridians required; and ſo for the reſt, as in the Triangular Quadrant.

Uſe XXVI.

To Draw the Dials.

1. *To draw a Horizontal Sun-Dial*, Fig. I.

FIrſt draw a Meridian line for 12 a Clock, as A B, and in that line deſign a point for a Center as at C, through which point draw a perpendicular line for 6 and 6, alſo draw two lines at pleaſure, parallel and equidiſtant from 12, as D E and F G. Then by *Uſe* XXIII. *Rule* 3. count C D Radius, and to it find the Secant of the latitudes complement, *viz.* 38,30, thus, Take the extent C D between your Compaſſes, and ſetting one point in 51,30, the complement of 38,30, on the general

Scale of Sines, lay the Thred to the neareſt diſtance, then take the neareſt diſtance from 90 to the Thred, and that ſhall be the Secant of 38,30, to lay from D to E, and from F to G, and draw the line E G.

Then make the extent B E, or B G, a parallel in 90, and lay the Thred to the neareſt diſtance, then take the parallel extents, from the points in the general Scale, being Tangents for the Hours and Quarters, and lay them from B toward E and G, for the Hours and Quarters required. Plainly thus;

Take the extent E B between your Compaſſes, and ſetting one point in 90, with the other lay the Thred to the neareſt diſtance, and there keep the Thred, by noteing what deg. and m. is cut by it, when right laid, then the like parallel extent from the ſeveral hour points, to the Thred at neareſt diſtance, ſhall be the ſeveral Tangents of the Hours and Quaters

Quarters required : then having marked the points for 10,11,1, and 2, from 12 both ways, take the extent D E, and make that a parallel Tangent of 3 hours or ſine 90 as before, and lay the Thred to the neereſt diſtance, and note it, and ſo keep it till you have taken all the points ſeverally, and laid them one after another, on both ſides from D and F, at 6 and 6, for 4,5,7, and 8, in the morning, and for 4,5,7, and 8 at the afternoon, them, and the quarters between if you pleaſe.

Note that the ſame manner of working, and the ſame points, ſerves for all erect Decliners, therefore mind this well, for I intend brevity in the reſt of the Dyals.

The height of the Stile will be 51 gr. 30 minutes, or equal to the Latitude at all places, for a *Horizontal Dyal*.

2. *To draw a South erect Dyal*, Fig. II.

As in the former Horizontal Dyal,

your firſt line was a Meridian line, ſo here in this Dyal (the Plain being ſuppoſed fixed) a Perpendicular line, in the middle of the plain is firſt to be drawn for the hour of 12, in which line deſign a point for the Center, and in that Center croſs the former perpendicular line, with a Horizontal line, for the two 6 a Clock hour lines, as the two lines A B. and 6 and 6 demonſtrate. Alſo, draw the lines 6 9 and 6 3, parallel to A B. Then mark A 6, a parallel ſine of 38,30, the complement of the latitude, and lay the Thred to the neareſt diſtance, then take out parallel 90, and lay it from 6 to 3, and from 6 to 9, and draw the line 9,3 for the lower Co-tangent line; then make 6 A, a parallel in 90, laying the Thred and noteing it and take out the hours as before in the Horizontal, and lay them both wayes from B for the noon-hours.

Again, make 6,9 or 6,3 a parallel

ſine 90 as before, and lay them, viz. the hour points, from 6 on both ſides downward only, for the morning and evening hours, for the Stile take parallel 38,30, from the general Scale, and ſetting one point of the Compaſſes in B, with the other make the touch of an arch, as at C, and draw the line A C for the Stile line of the Dyal, 38,30.

3. *To draw an Erect Direct North Dyal*, Fig III.

As before draw a perpendicular line, as A B, and a Horizontal line, as 6,C,6, alſo draw two lines at pleaſure equidiſtant from, and parallel to, A B, as 4,8 and 8,4; then make C 6, a parallel ſine or the co-latitude 38,30, than take out the points for the hours, and lay them on both ſides, from 6 to 7 and 5, and from 6 to 8 and 4, upwards and downwards, as in the Figure for the Stile, make any diſtance you pleaſe, as C A, a parallel ſine of 90, then take out parallel 38,28, and make a touch of an arch, as at D, then draw C D for the Stile, which in the North Dyal muſt point upward, as it did downwards in the South Dial, and right over the line A C.

4. *To Draw an Eaſt or Weſt Erect Dial.*

Rectifie the Rule by putting in the looſe piece, then put a plummet on the Thred, and apply the Rule to the Wall, and cauſe the Thred playing well to cut 38,30 the complement of the latitude, then by the head Leg draw the lines A B, C D, parallel one to another, for the two Contingents or Equinoctial lines.

Then in thoſe lines deſign a point where you intend 6 a Clock ſhall be, and in that point croſs the former at right angles, and draw that line for 6 a Clock hour line, then reſolve with your ſelf how far you intend the extent to be from 6 to 11, or from 6 to 10, what you pleaſe; take the ſame extent between your Compaſſes, and lay a Rule or Thred to the Center, at 60 in the looſe piece, and put one point of the Compaſſes in 75, on the looſe piece; then put from you, or draw near you, the Rule, till the other point of the Compaſſes will but juſt touch it, then there ſo keep it fixed, then the neareſt diſtance from 60, on the Tangent on the looſe piece, to the edge of the Rule, ſhall reach from 6 to 10, on both the equinoctial lines firſt drawn; again the extent from 45 on the looſe piece, to the Rule, ſhall reach from 6 to 9; the extent from 30 to the Rule, ſhall give the meaſure from 6 to 4 and 8; laſtly the neareſt diſtance from 15 to the edge of the Rule (cuting the Center at 0,60) ſhall be the extent from 6 to 5 and 7; then by theſe points, lines drawn parallel to 6, or perpendicular to the equinoctial,

equinoctial , ſhall be the hour line required : the Stiles elevation muſt always be the ſpace of 3 hours from 6, viz. the meaſure from 6 to 9 in the Eaſt Dial , or from 6 to 3 in the Weſt, and ſtand right over and parallel to 6 and the plain.

Note, That the Weſt Dial is like the Eaſt ſide , when you turn the backſide of the Paper toward you, and looking againſt the light, and the moſt convenient way to draw the equinoctial line is , to let the plummet play on 81,30 , and then draw the line by the looſe piece , for an equinoctial line , and 6 perpendicular to it, and all the reſt as before. Leaſt I ſhould be obſcure in laying the Rule to the Center at 60 , mind this Figure VI, let A B repreſent the Tangent line on the looſe piece , numbred 15,2045,60,75 at B , then let A C repreſent the edge of a Rule laid over the Center at **A** , and as far of 75 at AB , as the intended meaſure , from 6 to 11 , *viz.* from B to the arch DE.

Then the Rule ſo laid, keep it ſo, and the neareſt diſtances from 60 45, 30 15, to the edge of the Rule A C, ſhall be the hour ſpaces required, to lay from 6 on the equinoctial lines.

5. *To draw an Erect Declining Dial*, Fig. VII.

Let the declination of an erect declining Dial be 20 South Eaſt; then firſt by *Uſe* XXV. find the requiſites, as ſubſtile, ſtile, 6 and 12, and the inclination of Meridians, and ſet them down in a Paper in this manner, or how elſe you think fit.

Subſtile	15	12
Stile	35	45
12 and 6	66	42
Incli. Meri.	24	50

Then firſt draw a perpendicular line for 12, as A B, and then two lines on each ſide, equidiſtant and parallel to 12, as C D and E F; then appoint a place for a Center, as at A, and from thence deſign any diſtance, as to B, and take it between the Compaſſes, and make it a parallel ſine of 90, and lay the Thred, and note the place or ſo keep it; then take out the parallel ſine of 15, 12, and ſetting one foot in B, deſcribe the touch of an arcke, as at G, and draw A G for the Subſtile Weſtwards; then make A G equal to A B, and the Thred laid as before, take out the parallel ſine of 35,45, and ſetting one point in G, draw the touch of an arch, as at H, and draw A H for the Stile, then take out the parallel ſine of 66,42, and draw the touch of an arch, as at I, one point being fixed in B, and draw that line for 6; or you may lay theſe angles off in an arch, as with Chords by *Uſe* XXIII. *Rule* 5.

Then take the diſtance of the parallel A C, and by *Uſe* XIII. *Rule* 4. make it a parallel ſine 90, then take out the parallel ſine of 70, the complement of declination, and make tha a parallel ſine of 38 28, and then take out parallel 90, and lay it from A to K, and on the parallel from 6 to D or 9, then make K D a parallel ſine 90, and lay off the hour points both ways from K, for 10,11, 1 and 2, and if you take the Tangent of Inclinations from 60,0, and lay it on the general Scale downward and then take from thence to the Thred it will reach from K to G, the ſubſtile, if you have made no miſtake in your former work.

Laſtly, Make 6 D a parallel ſine of 90, and lay off the hour points from 6, as you did before, and you have points to draw the hour lines withal; if you want a point at 4 after-noon, the meaſure from 9 to 8 on the Eaſt ſide, is equal to the meaſure from 3 to 4, and ſo for all others, as 5 or 6.

6. *To draw a North West Erect Decliner, declining* 30 *deg.* Fig. VIII.

Firſt, As before by Uſe XXV. nd the requiſites and ſet them down eady for your uſe.

Subſtile	21	40	*North Weſt* 30 *d.*
Stile	32	35	
12 and 6	57	49	
Incli. Mer	36	25	

Then draw a perpendicular line, repreſenting 12 at midnight, as A B, and a parallel thereto, as C D, on both ſides, if you pleaſe as before, then being a North plain, the Stile looking upwards, the Center is downward, as at E.

Alſo Note in this, And all other plains the Subſtile is laid contrary to the coaſt of declination, lay off, as is taught before, the Subſtile, Stile, 12 and 6.

Then mak: A C a Secant of 30 and take out the Secant of 51,30 and make it a parallel ſine of 90, an lay down the hour points as befor and draw the hour lines that be proper for ſuch a declining Dial, to w thoſe that can come in uſe at an time of the year.

7. *To draw a far Declining Dial* Fig. IX.

Thoſe Dials that decline above 4 or 50 deg. ought to have their Stile augmented, or elſe they will no be comely nor convenient, which augmenting may be done ſevera ways, but this by the Sector I conclude moſt neat and ready in operation.

As for *Example* in a South-eaſt decliner 80 deg.

Firſt, as before, find the requiſites and ſet them down; then by help of the inclination of Meridians, make a able of hour arkes at the pole, by lling the inclination of Meridans always 12; ſubſtract 5 deg. for an Hour, ,30 for half an our, and 3 deg. 5 m. for a quarter, as often as you an, ſetting down he remainder as ere you ſee.

Subſtile	38	02
Stile	06	12
6 and 12	38	53
Inc. Meri.	82	08
	74	38
11	67	08
	59	38
10	52	08
	44	38
9	37	08
	29	38
8	22	08
	14	38
7	07	08
	00	22
6	07	52
	15	22
5	22	52
	30	22
4	37	52
	45	22

Then when you an take no more, ke the remainder from 7,30, and t down what emains, and then dd 7,30 till ou have enough s in the Table nexed.

Then firſt draw perpendicular ne on the North dge of the plain as A B, and from

B

B lay off 38,2, for the Subſtile, be done by Tangents or ſines by C H 2,2 then draw that line A C, and croſsi with two lines ſquare to it, as D E F G; in the upper line deſign a plac for 11, and 3 ½ as E and D, then tak 45,22, the number in the Table, fo 3 ½ out of the Tangents on the looſ piece, and add it to the Tangent o 67,8, for 11, and the Sum on the ſam Tangent line is 73,35, then take th extent D E, and ſetting one point i 73,35, lay a Rule over the Cente at 60,0 on the looſe piece, and th neareſt diſtance as in the Eaſt Dial then take out the parallel Tangent o 45,22, and lay it from D, and th parallel Tangent of 67,8, and lay i from E, and they meet in H, th place of the Subſtile, then draw H I parallel to A C for the Subſtile, and then lay off all the numbers in th Table, for the hours and halfs, as th Rule lies in its firſt poſition, and mar them with ſmall figures to know them again, and be ſure to draw the touch of an arch with the Tangent of 45 deg. for the Stiles height, as H K.

Then make H I a parallel Tangent of 45, and take out the Tangent of 6,12 for the Stiles height, and lay it from I to L, then take out H K the firſt Radius, and lay it from L to M in the touch of an arch, then draw K M by the convexity of the two arches, for the Stile line; then make I M a parallel Tangent of 45, by laying a Rule to 60,0 on the looſe piece and 45, by the extent I M, then from I lay off all the numbers in the Table again, and mark them as before on this other contingent line G F, as in the figure.

Then lines drawn to thoſe marks, ſhall be the hour lines for a South-eaſt declining 80 deg. the Stile muſt be according to the pattern in the figure, *viz.* the line N O ſet right, and ſo far over H, as the diſtance in the Dial expreſſeth.

If this ſeem too brief, you have it

more plainly and copiouſly in the *Triangular Quadrant*, to which I refer you, for the making all ſorts of Dials, and for any Latitude and the Ornaments alſo.

Uſe XXVIII.

To find Altitudes, or Diſtances by the Quadrat and Shadows, and firſ for Acceſſable, at cne Station by the Shadows

1 SEt the Rule to an Angle of 6 deg. by putting in the looſ piece, and hang a plummet on th Thred; and looking up to the mar at E, Suppoſe the plummet line fal on 1, or go nearer or further ti it fall on 1, on the line of ſhadow then conclude that the altitude of th

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Object at E , is juſt as high above the level of the eye, as from C to A, *viz.* the place of ſtanding, to right under the Object at A.

Alſo if you go ſo far off the Object that the line falls on 2 of contrary ſhadows, *viz.* at 26, 34, the Object is but half the length of the diſtance A D, but if you approach ſo near as that the Thred ſhall fall on 2 of right ſhadow, at 63,26, the height is double the diſtance, as A E is twice A B; but to find the height at any obſervation by the line of Numbers, ſay, always for right ſhadow, that is, when the Thred falls between 1 at 45 and 90, As 1 to the parts cut by the Thred, ſo is the meaſured diſtance to the height required; but for contray ſhadow, that is when the Thred falls between 0,60 on the looſe piece, and 45, then ſay, As the parts cut by the Thred are to 1, ſo is the meaſured diſtance to the height.

Example, The extent from 1 to 2, the parts cut by the Thred at B, ſhall reach the ſame way on the line of Numbers from 15, the meaſured parts B A to 30, the height required.

Again, The extent from 2, the parts cut at D to 1, ſhall reach the ſame way from 60, the parts meaſured, to 30 the height as before.

The ſame Rules ſerves at any odd parts whatſoever, and the ſame manner of work ſerveth; if you meaſure the ſhadow of any Tree or other Object, and obſerve what part of the line of ſhadows the Thred falleth on, in taking the Suns Altitude.

2. *For an Unacceſſable Altitude at two Stations by the Line of Shadows.*

Firſt, Obſerve at B, and note the parts cut, *viz.* 2, ſecondly, obſerve at C, and note the parts cut ſuppoſe

1, or any other odd part; but to obſerve more exactly, uſe the deg. which are cloſe divided, and ſet down both the Angles, *viz.* the Angle obſerved and his complement, counting one as figured, and the complement (*viz.* the angle at the top) count from the head; as here in this Obſervation, the obſerved Angle is at C 45, ſo likewiſe is the complement 45, counting from the head alſo.

The other Angle obſerved at B is 63,26; his complement counting from the head is 26,34, the Angle at D is 26,34, but his complement at E is 63,26, and the ſame Rule for all right Angle Triangles as theſe are.

Then obſerve the difference in Tangents between the two Angles at the top found thus, the complement at the Angle at B, *viz*, A E B is 26,34, the Thred laid to 26,34 on the Quadrat, gives 50; again, the Thred laid to 45 on the Quadrat, gives 100 ; then 50 taken from 100, the difference is 50.

Then the Rule is by the line of Numbers onely.

The extent from 50 the (laſt found) difference to 100, ſhall reach from 15 the meaſured diſtance between B and C, to 30 the height.

Again, in the other Stations, *viz.* at C and D, the Angle at the Top is at C 45, the Angle at D at the Top is 63,26, *viz.* the complement of the Angle at D, then the Thred laid to 45, on the Quadrat, gives 100, the Thred laid to 63,26, on the line of ſhadows, gives 200, Subſtraction made remains 100, for the difference in Tangents.

Then as before, the extent from 100 to 100, ſhall reach from 30 the meaſured diſtance to 30 the height; and the like for any odd num- whatſoever.

Or elſe without the difference in Tangents by ſines and lines, ſubſtract the complemeut of the Angle at C, from the complement of the Angle at D, *viz.* 45 from 63,26, and the difference is 18,26, then ſay,

As lateral 30, taken from any Scale of equal parts, as inches, foot meaſure, or lines, is to the parallel ſine of 18,26, the oppoſite angle, laying the Thred to the neareſt diſtance; ſo is the parallel ſine of 26,34, to 42,45, the ſide CE.

Again, Secondly, As lateral 42,45, to parallel ſine 90, ſo is the parallel ſine of 45 to 30 the altitude required.

The like work at two operations ſerves any right line Triangle, but much more of this kind in the *Triangular Quadrant*.

Note, That one good way to come by an altitude at two ſtations, is to go too and fro, till the Angle at the fartheſt ſtation be half the Angle at the neareſt, as the Angle

B is double the Angle at E, an the meaſured ſide B F , is equal t B E; then as 90 to B E, ſo is 63,26 to A F 30.

3. *To take the meaſure of a lon Diſtance*, Fig XI.

Let A be the ſtation to ſtand at, and E the mark a far off, whoſe diſtance is to be meaſured.

Set the Inſtrument, *viz.* the Triangular Rule , on ſome flat ſteady place at A, , and look exactly to E, then turn the index about in a perpendicular line to A E, toward B, and meaſure in that line any number of yards or feet, as ſuppoſe 70, and there leave a mark at B, there alſo get the Angle A B E , and by conſequence the Angle B E A , the complement thereof.

Then meaſure from A towards E, any certain number of yards, as ſuppoſe 100, and then from thence g

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toward D , in a ſtrait perpendicular line to A E, till you eſpy E and B in a right line ; then meaſure alſo this diſtance C D , as ſuppoſe 58 , then are you furniſhed two ways to get the diſtance C E or A C, as followeth.

1. *Firſt , by the line of numbers onely.*

As the difference between A B and C D 12 , is to the diſtance between A and C 100, ſo is the meaſured diſtance A B 70 , to the diſtance required A E 55 ; ſo alſo is C D 58 to C E 485.

2. *Secondly , To work the ſame by ſines and lines.*

As lateral 70 , from any ſmall Scale, to the parallel ſine of 6 gr. 48 m. ſo is the parallel ſine of 83,12 , to the lateral length 585 , on the ſame Scale you took 70 from; and ſo is the parallel ſine of 90, to E B 590.

Note, That you ought to meaſure A B very exactly, and take the angle A B E exactly alſo, or elſe your work will never agree, in the two ſeveral ways of working.

4. *To find any Breadth or Diſtance by Scale and Compaſs or Calculation*, Fig. XII.

Let A B be two marks, as the two corners of a Wall, and let the meaſure A B be demanded, and their diſtance from C and D, the two ſtations.

Firſt, Set up a mark at one ſtation, and meaſure any certain meaſure as you pleaſe, and which way you find moſt convenient, as at C and D 100 foot between.

Set up the Rule on his ſtaff right over C, and ſet the lines and ſines directly to D , the other mark, and there fix it then direct the ſight exactly to B , and note the Angle that you find, as here 45 deg. then direct the ſighs to A , and note that Angle from D 113. deg.

Secondly , remove the inſtrument to D , and ſet the ſines and lines exactly to C, then fix it there, and then obſerve the Angles C D A 42,30, and C D B 109.

Then the Three Angles of every Triangle being equal to 180 deg. the ſum of 42,30 and 113 , *viz.* 155,30, taken from 180 remains 24 30, the Angle D A C, likewiſe the ſum of 45 and 109, *viz.* 154 taken from 180 reſts 26, the Angle C B D.

Then by theſe obſervations , you may with Scale and Compaſs on Paper, or ſlate , draw the figure thus , as annexed.

Draw the line C D, and from your Sccale lay down 100 parts, from C to D, then with a line of Chords as the azimuth line , the general an particular Scale of ſines are , makin the ſine 30 Radius , as by Uſe 2 Rule 5.

And lay off the Angles as obſerved , and draw the lines as in th Figure, then you may meaſure ever ſide and diſtance , as you pleaſe, b the ſame Scale you took C D fro

As for the Calculation , it is be performed by the Tables of Sines an Tangents , or a *Gunters Rule* is mor ready than the natural lines , an for ample directions therein , conſult the *Triangular Quadrant* an Doctrines of Plain Triangles.

Uſe XXIX.

To find the Hour of the Day and Suns Azimuth in any other Latitude by the general Scale of Sines.

That the Rule may be general for more places than one, I have added this Uſe.

THerefore firſt to find the Latitude of any place, do thus, Juſt at noon on any day, find the Suns Altitude as carefully and exactly as you may, which to do the more certainly, obſerve the Altitude a little before noon, and ſet it down in a Paper or a Book , and then continue ſo obſerving every minute, or as often as you can , ſtill ſetting every Obſervation down , till you are ſure it is paſt noon , then the comparing of them together , will confirm you which is the beſt or trueſt Obſervation , and the greateſt and certaineſt of them you may conclude to be the Suns Meridian Altitude for that day : To which Meridian Altitude in Winter or Southern Declinations, add the Suns Declination . and the Sum is the complement of the Laltitude.

But in Summer time or Northern Declinations , ſubſtract the Declination out of the Suns Meridian Altitude , and the remainder is the complement of the Latitude required.

Example , Suppoſe on the 10th. of *April* the Noon Altitude of the Sun is 50 deg. 13 m. the Declination of the Sun the ſame day is 11 deg. 45 m. Northward , then 11 deg. 45 m. taken

ken from 50 deg. 13 m. reſts 38 deg. 28 m. the complement of the Latitude at *London*.

Again , Suppoſe on the 20th of *February* the Suns Declination is 70 deg. 3 m. South , and his Meridian Altitude the ſame day 31 deg. 25 m. the ſum of them added together makes 31 deg. 28 m. the complement of the Latitude , which taken from 90 deg. reſts 51 deg. 32 m. the Latitude of *London* required. This Rule ſerves from 23 deg. 30 m. to 66 deg. 30 m. North Latitude.

1. *Then the Latitude, Suns Declination, and Altitude being given, to find the Hour of the Day.*

In the Equinoctial , when the declination is 0 , take the lateral ſine of the Suns Altitude from the general Scale , and make it a parallel in the co-ſine of the Latitude , and with the other point lay the Thred to the neareſt diſtance, and on the deg. it gives the hour required, counting 90 at the head, 6 and 60,0, on the looſe piece 12.

Example, At 20 deg of Altitude in 51 deg. 30 m. Latitude, you find 8 hours 12 m. in the morning or 3 hours 48 m. in the afternoon.

2. *To find the Hour at any other time of the year.*

Count the Suns Declination on the deg. from 90 at the head toward the looſe piece, and there lay the Thred.

Then the neareſt diſtance from the ſine of the Latitude, counted on the general Scale, to the Thred, ſhall be the ſine of the Suns Altitude at 6, which diſtance in Summer lay from the Center downward, and in Winter from the Center (where the Thred is faſtened) upward, and note that place for all that day.

Then take from that noted place to the ſine of the Suns Altitude on the general Scale, with this diſtance ſet one foot in the co ſine of the Latitude, and lay the Thred to the neareſt diſtance and ſo keep it.

Then take the neareſt diſtance from the ſine of 90 to the Thred, then with this diſtance ſet one foot in the co-ſine of the Suns declination, and with the other lay the Thred to the neareſt diſtance, and in the deg. it cuts the hour required.

Example, Suppoſe on the 5th of *April* or on the 12th. of *February*, when the Suns Declination is 10 deg. and the Suns Altitude on each day 15 deg. what is the hour?

The Thred laid to 10 on the deg. counting from the head, and the neareſt diſtance from 51 deg. 30 m. to the Thred, gives the ſine of 7 deg. 45 m, the Suns height at 6 in *April*, or his depreſſion at 6 in *February*. This diſtance laid downward reaches to 7 deg. 45 m. on the large general ſcale, for the 5th. of *April*, o being laid upward reaches to near 2 on the ſmall ſines, for the 12th. of *Fe-bruary*.

There the extent from 7 deg. 45 m the noted point to 15, the Suns preſent altitude, taken between the Compaſſes, ſet one point in 38 deg. 30 m th latitudes complement, and with th other lay the Thred to the neareſt diſtance; laſtly, take the neareſt diſtance from 90 to the Thred, and ſetting one point in 10 deg. counting from 90, (or the ſine of 80, the complement of 10) and lay the Thred to the neareſt diſtance; and on the deg. it gives 46 m. after 6 in the morning or 14 m. after 5 at night, on the 5th of *April*. But on the 12th. of *February* the lateral extent from the upper mark (near 24 deg. on the ſmall ſines) to 15, laid from 38,30, and the thred laid to the neareſt diſtance: Laſtly, the neareſt extent from 90 to the thred, laid from the ſine of 80 the complement of the Suns declination, and the thred laid to the neareſt diſtance again, gives 8 and 40 min. in the morning, or 3 and 20 m. in the after-noon, for the 12th day of *February.*

Which work is only an adding of the Suns depreſſion at 6 in Winter to the Suns preſent altitude; or the ſubſtracting the Suns height at 6 from the preſent altitude in Summer, for the firſt operation.

3. *The Suns altitude and declination and latitude given, to find the Suns Azimuth generally.*

Take the ſine of the Suns declination from the general Scale, ſet one foot in the ſine of the latitude, on the ſame Scale, and with the other lay the thred to the neareſt diſtance, and on the degrees it gives the Suns altitude at

Eaſt or Weſt ; count this on the gen ral Scale, and take from thence to th Suns preſent altitude , which wo makes a ſubſtraction of the leſſer fro the greater.

With this remaining diſtance ſet on foot in the co-ſine of the latitude , an lay the thred to the neareſt diſtance then take the neareſt diſtance from th ſine of the latitude to the thred : ſ this again in the co-altitude , and th thed laid to the neareſt diſtance , give the Suns azimuth from South required.

Example , In latitude 51 deg. 3 min , the declination 7 deg. North, th Suns Vertical altitude at Eaſt will b 8 deg. 57 min. and the preſent altitud being 30, the reſidue or difference wi be the ſine of 20 deg. 13 min. and th Suns azimuth found thereby will be 6 deg. 12 min. from the South.

4. *But in Winter time or Southern declinations, firſt find the Suns amplitude thus :*

Take the lateral ſine of the Suns de-
ination, make it a parallel in the co-
e of the latitude ; then the thred
id to the neareſt diſtance in the de-
rees, gives the Suns amplitude ; which
ou muſt remember.

Then take the lateral ſine of the
ns preſent altitude, and make it a pa-
llel in the co-ſine of the latitude, and
y the thred to the neareſt diſtance,
en the neareſt diſtance from the ſine
f the latitude to the thred, is to be
dded laterally to the ſine of the ſuns
mplitude, and that ſum taken between
our Compaſſes ; make it a parallel
ne in the co-ſine of the ſuns preſent
titude ; and the thred laid to the
areſt diſtance on the degrees, gives
e ſuns preſent azimuth from the ſouth
quired.

Example, At 15 degrees of declination and 15 degrees of altitude, th amplitude is 24 deg 40 min. and th azimuth from ſouth 39 and 30; bu from Eaſt or Weſt 50, 30.

Much more variety of th kind may you find in the Triangular Quadrant, *chap.* 15.

FINIS.

THE CONTENTS OF THE PARTICULARS

Contained in the Preceding

SUPPLEMENT.

A Short Deſcription.

2. *The Uſe of Inches and oot Meaſure together, in Reduction, d the rate of* 1, *to find the price of* 00, *and the contrary.*

3. *The price of* 272 *foot* ¼, *or a od, at any rate* per *foot, and the ntrary.*

4. *The price of* 1 *given, to find the ice of* 100, *and the contrary, by mbers and pence annexed, by inſpe- ion ſeveral wayes.*

5. *A further Uſe of the Numbers in Multi-*

Multiplication, Diviſion, Reduction, Rule of Three, Square and Cube Roots, Practice in Domeſticks, Board, Timber, Square and Round; Meaſuring Bric Work, and Reducing at one Work.

6. *To find the Rods and odd Feet a one extent of the Compaſſes.*

7. *To find the price of all or an part.*

8. *Of Intereſt both Simple and Compound.*

9. *The Uſe of the Thirty and Fort Scales.*

10. *The Uſe of the Line of Circles*

11. *The Uſe of the Line of Chords*

12. *The Uſe of the Line of Tangents*

13. *The Uſe of the Quadrant ſide.*

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